Lecture 23: Logic I

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Announcements

Roadmap

Introduction

Functions

Data

Mutability

Objects

Interpretation

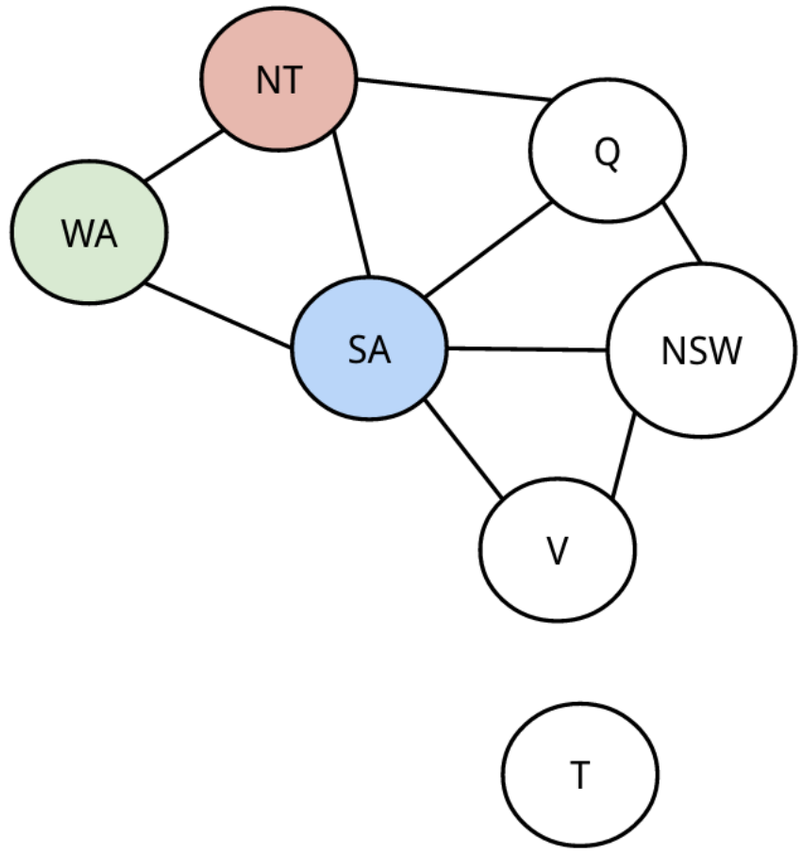
Paradigms

Applications

- This week (Paradigms), the goals are:
  - To study examples of paradigms that are very different from what we have seen so far
  - To expand our definition of what counts as programming

Today's Example: Map Coloring

- Problem: Given a map divided into regions, is there a way to color each region red, blue, or green without using the same color for any neighboring regions?



Imperative Programming

- All of the programs we have seen so far are examples of *imperative programming*, i.e., they specify detailed instructions that the computer carries out
  - In imperative programming, the programmer must first solve the problem, and then code that solution
  - But what if we can't solve the problem? Or what if we can't code the solution?

```
# Imperative map coloring
colors = ['red', 'blue', 'green']
for region in map:
    i = 0
    while not region.valid:
        region.color = colors[i]
        i += 1
        if i >= len(colors):
            # ???
```

Declarative Programming

- In *declarative programming*, we specify the properties that a solution satisfies, instead of specifying the instructions to compute the solution
  - We tell the computer *what the solution looks like*, instead of *how to get the solution*
  - This is simpler, more natural, and more intuitive for certain problems and domains
- We will write code that looks like this:

```
# Declarative map coloring idea:
Find a solution where:
- All regions of the map are colored
- No neighboring regions have the same color
```

Disclaimer

- Declarative languages move the job of solving the problem over from the programmer to the interpreter
- However, building a problem solver is hard! We don’t know how to build a *universal problem solver*
- As a result, declarative languages usually only handle some subset of problems
- Many problems will still require careful thought and a clever approach from the programmer
- Think *declaratively*, not *imperatively*

Today’s Lecture

- Solve some cool problems
- As long as the problem is not too big
- Requires cleverness from the programmer

Most Declarative Programming

- Solve less cool problems
- But the problems can be much bigger
- More standard approach for programmers

Logic

The programming language

Logic (demo)

- The Logic language was built for this course
- Borrows syntax from Scheme and semantics from Prolog (1972)
- Programs consist of *relations*, which are lists of symbols
  - Logic is pure symbolic programming, no concept of numbers or arithmetic of any kind
- There are two types of expressions:
  - *Facts* declare relations to be true
    - All relations are false until declared true by a fact
  - *Queries* ask whether relations are true, based on the facts that have been declared
    - It is the job of the interpreter to figure out if a query is true or false

Variables

- Relations can contain variables, which start with ?
- A variable can take on the value of a symbol

```
logic> (fact (border NSW Q))
logic> (query (border NSW Q))
Success!
logic> (query (border NSW NT))
Failed.
logic> (query (border NSW ?region))
Success!
region: q
```

variable

- Relations in facts can also contain variables

```
logic> (fact (equal ?x ?x))
logic> (query (equal brian brian))
Success!
logic> (query (equal brian marvin))
Failed.
```

Negation (demo)

- What if we want to check if a relation is false, rather than if it is true?
  - (not <relation>) is true if <relation> is false, and false if <relation> is true
  - This is an idea known as *negation as failure*

```
logic> (query (not (border NSW NT)))
Success!
logic> (query (not (equal brian marvin)))
Success!
logic> (query (not (equal brian brian)))
Failed.
```

- Sometimes, negation as failure does not work the same as logical negation
  - It is useful to be able to understand the differences

```
logic> (query (not (equal brian ?who)))
Failed.
```

Compound Facts (demo)

- Compound facts contain more than one relation
- The first relation is the *conclusion* and the subsequent relations are *hypotheses*

  `(fact <conclusion> <hypothesis-1> ... <hypothesis-n>)`

- The conclusion is true if, and only if, all of the hypotheses are true

```
; declare all border relations first
logic> (fact (two-away ?r1 ?r2)
             (border ?r1 ?mid)
             (border ?mid ?r2)
             (not (border ?r1 ?r2)))
logic> (query (two-away ?r1 ?r2))
Success!
r1: nsw   r2: wa
r1: nt    r2: v
r1: q     r2: wa
r1: q     r2: v
```

An Aside

```
logic> (query (border NSW Q))
Success!
logic> (query (border Q NSW))
Failed.
```

- Relations are not *symmetric*, which is weird for borders
- We can fix this by declaring more facts for borders, but we won’t do that yet because doing so introduces *cycles*
- Handling cycles is hard (remember cyclic linked lists?), and makes the whole example a bit too complicated
  - So we will leave it out for now
- But the basic idea is that, if we have cycles, we have to keep track of what regions we have already seen, to make sure we don’t look through the same regions forever

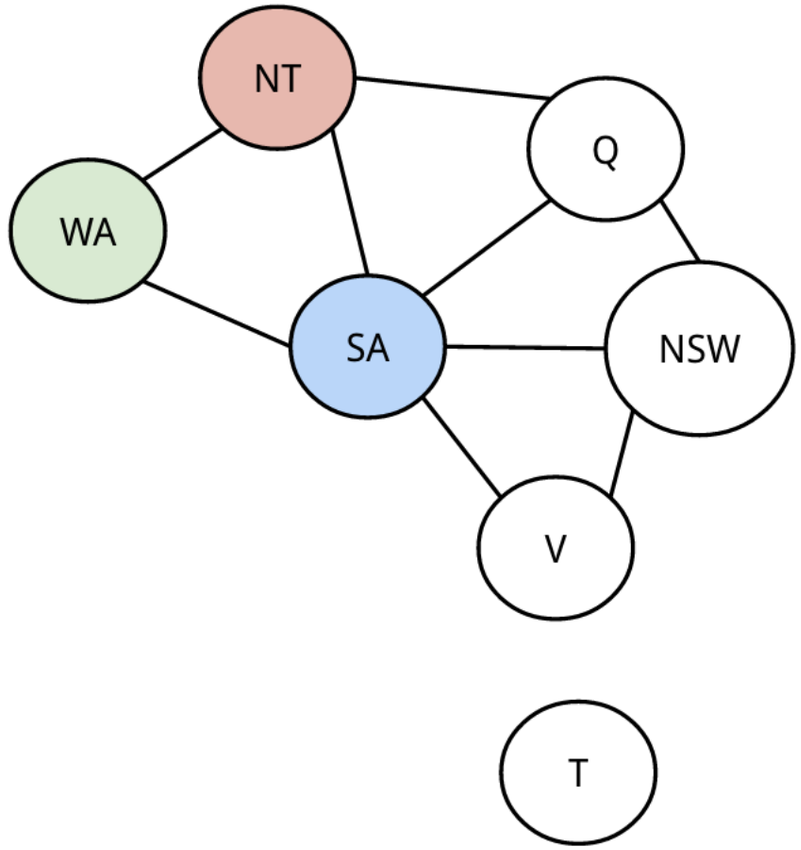
Compound Queries

- Compound queries contain more than one relation

  (query <relation-1> ... <relation-n>)

- The query succeeds if, and only if, all of the relations are true

```
logic> (query (two-away NSW ?region)
              (two-away Q ?region))
Success!
region: wa
logic> (query (two-away ?r1 ?r2)
              (border NT ?r2))
Success!
r1: nsw   r2: wa
r1: q     r2: wa
```



Recursive facts

Also, hierarchical facts

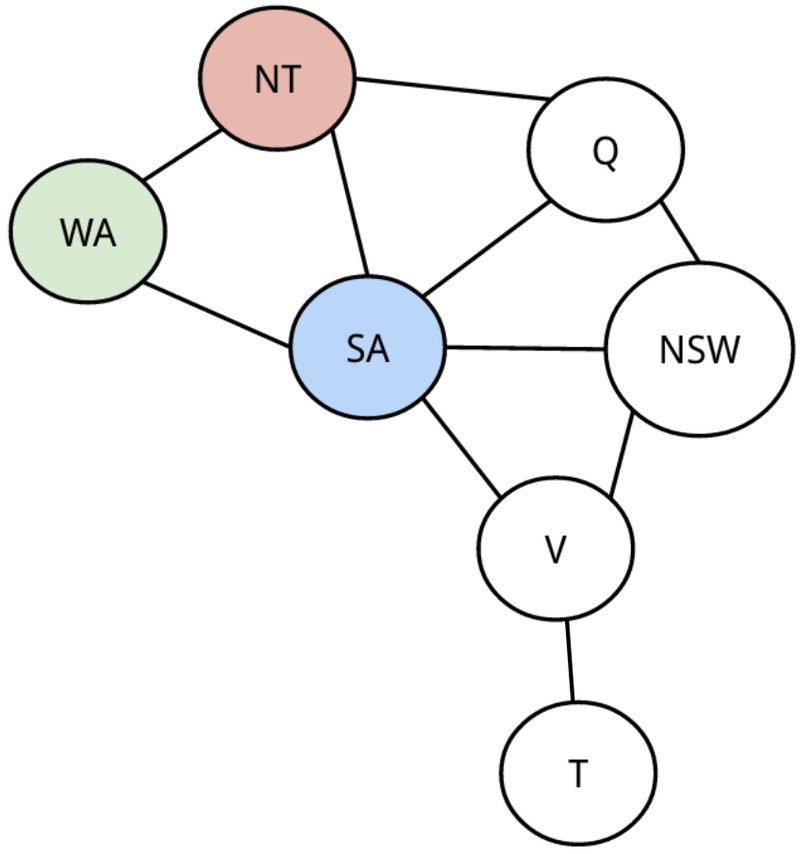
Recursive Facts (demo)

- A *recursive fact* uses the same relation in the conclusion and one or more hypotheses
- Just like in imperative programming, we need a *base fact* that stops the recursion

```
logic> (fact (connected ?r1 ?r2)
             (border ?r1 ?r2))
```

```
logic> (fact (connected ?r1 ?r2)
             (border ?r1 ?next)
             (connected ?next ?r2))
```

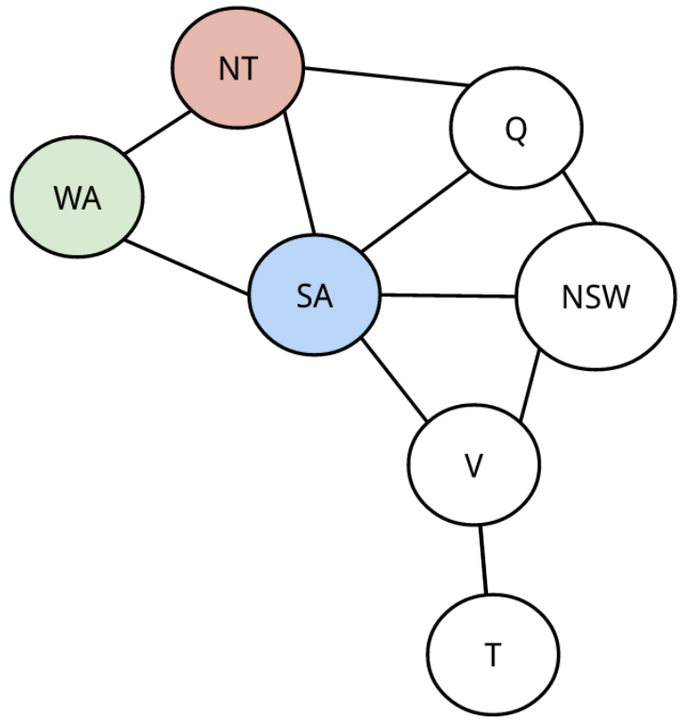
```
logic> (fact (border V T))
logic> (query (two-away NT T))
Failed.
logic> (query (connected NT T))
Success!
```



Recursive Facts

- The Logic interpreter performs a search in the space of relations for each query to find satisfying assignments

```
logic> (fact (connected ?r1 ?r2)
             (border ?r1 ?r2))
logic> (fact (connected ?r1 ?r2)
             (border ?r1 ?next)
             (connected ?next ?r2))
logic> (query (connected NT T))
Success!
```



(border V T) ⟹ (connected V T)

(border SA V) ⟹ (connected SA T)

(border NT SA) ⟹ (connected NT T)

Hierarchical Facts

- Relations can also contain lists in addition to symbols

```
(fact (australia (NSW NT Q SA T WA V)))
```

symbol — australia

list of symbols — (NSW NT Q SA T WA V)

- The fancy name for this is *hierarchy*, but it's not a fancy or complex idea

- Variables can refer to either symbols or lists of symbols

```
logic> (query (australia ?regions))
Success!
regions: (nsw nt q sa t wa v)
logic> (query (australia (?first . ?rest)))
Success!
first: nsw   rest: (nt q sa t wa v)
```

- Why the dot? Because we are using Scheme lists,
  `(nsw nt q sa t wa v)` is the same as `(nsw . (nt q sa t wa v))`

first — nsw

rest — (nt q sa t wa v)

Example: Membership (demo)

- Recursive and hierarchical facts allow us to solve some interesting problems in Logic
- As a first example, let's declare facts for membership of an element in a list

```
logic> (fact (in ?elem (?elem . ?rest)))
logic> (fact (in ?elem (?first . ?rest))
             (in ?elem ?rest))
logic> (query (in 1 (1 2 3 4)))
Success!
logic> (query (in 5 (1 2 3 4)))
Failed.
logic> (query (in ?x (1 2 3 4)))
Success!
x: 1
x: 2
x: 3
x: 4
```

Example: Appending Lists (demo)

- Let's declare facts for appending two lists together to form a third list

```
logic> (fact (append () ?lst ?lst))
logic> (fact (append (?first . ?rest) ?lst (?first . ?rest+lst))
             (append ?rest ?lst ?rest+lst))
logic> (query (append (1 2) (3 4) (1 2 3 4)))
Success!
logic> (query (append (1 2) (3 4 5) (1 2 3 4)))
Failed.
logic> (query (append ?lst1 ?lst2 (1 2 3 4)))
Success!
lst1: ()          lst2: (1 2 3 4)
lst1: (1)         lst2: (2 3 4)
lst1: (1 2)       lst2: (3 4)
lst1: (1 2 3)     lst2: (4)
lst1: (1 2 3 4)   lst2: ()
```

Let’s Color Australia

In two different ways

Map Coloring Way #1 (demo)

- Idea: Create a variable for the color of each region
  - We have to make sure each variable is assigned to one of the symbols `red`, `green`, or `blue`
  - Then, we have to make sure the variables for bordering regions are not equal

- We can pretty closely follow what we wrote at the beginning of lecture:

```
# Declarative map coloring idea:
Find a solution where:
- All regions of the map are colored
- No neighboring regions have the same color
```

Map Coloring Way #1

Find a solution where:

- All regions of the map are colored
- No neighboring regions have the same color

```
logic> (query (in ?NSW (red green blue))
              (in ?NT  (red green blue))
              (in ?Q   (red green blue))
              (in ?SA  (red green blue))
              (in ?T   (red green blue))
              (in ?V   (red green blue))
              (in ?WA  (red green blue))
              (not (equal ?NSW ?Q))
              (not (equal ?NSW ?SA))
              (not (equal ?NSW ?V))
              (not (equal ?NT  ?Q))
              (not (equal ?NT  ?SA))
              (not (equal ?NT  ?WA))
              (not (equal ?Q   ?SA))
              (not (equal ?SA  ?WA))
              (not (equal ?SA  ?V)))
```

Map Coloring Way #2 (demo)

- Solution #1 was simple and allowed us to directly follow our original idea
- However, it wasn’t an elegant or efficient solution
  - Lots of repetition
  - No separation between *data and program*
  - As a result, only works for this specific map
- Let’s look at a more complicated and clever solution
  - We will first declare our data, which is our map
  - We will then try and find assignments with no conflicts
  - This way, our program does not repeat itself, and will be general to any map!

Summary

- We learned about *declarative programming* today
  - A completely different programming paradigm where our programs specify what properties solutions should satisfy rather than how to find a solution
  - This allows us to solve some problems in an easier and more intuitive manner
- We learned Logic, a declarative language
  - Logic consists of facts, which declare relations that are true, and queries, which ask if relations are true
  - Recursive and hierarchical facts allow us to solve many interesting problems
- This is very different idea, so you'll have to practice!