Lecture 20: Scheme II

Brian Hou
July 26, 2016

Announcements

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Roadmap

- Introduction
- Functions
- Data
- Mutability
- Objects
- Interpretation
- Paradigms
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Introduction

Functions

Data

Mutability

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Interpretation

Paradigms

Applications

- This week (Interpretation), the goals are:
  - To learn a new language, Scheme, in two days!
  - To understand how interpreters work, using Scheme as an example

The **let** Special Form

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```
scm> (define x 1)
x
scm> (let ((x 10) (y 20))
       (+ x y))
30
scm> x
1
```

Tail Recursion

Factorial (Again)

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```
(define (fact n)
```

Factorial (Again)

```
(define (fact n)
  (if (= n 0)
```

Factorial (Again)

```
(define (fact n)
  (if (= n 0)
    1
```

Factorial (Again)

```
(define (fact n)
  (if (= n 0)
    1
    (* n (fact (- n 1)))))
```

Factorial (Again)

```scheme
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scm> (fact 10)
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```scheme
(define (fact n)
  (define (helper n prod)
```

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How? Eliminate the middleman!

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Example: Length

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(define (length s)
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(define (length s)
  (if (null? s) 0
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(define (length s)
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(define (length-tail s)
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```
>>> r = range(11111, 1111111111)
>>> r[20149616]
20160726
```

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`(car (cons 1 2)) -> 1`	`(car        (cons-stream 1 2)) -> 1`
`(cdr (cons 1 2)) -> 2`	
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`(car (cons 1 2)) -> 1`	`(car (cons-stream 1 2)) -> 1`
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(cons-stream 1 (/ 1 0))   -> (1 . #[promise (not forced)])
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    (int-stream (+ start 1))))
```

Infinite Streams (demo)

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(define (int-stream start)
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Recursively Defined Streams

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(define ones (cons-stream 1 ones))
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1 1 1 1 1 1 ...

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1 $\boxed{1 \quad 1 \quad 1 \quad 1 \quad 1 \quad \ldots}$

Recursively Defined Streams

```
(define ones (cons-stream 1 ones))

(define (add-streams s1 s2)
```

1 $\boxed{1 \quad 1 \quad 1 \quad 1 \quad 1 \quad \ldots}$

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1 1 1 1 1 1 ...

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(define ints
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1 1 1 1 1 1 ...

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  (cons-stream 1
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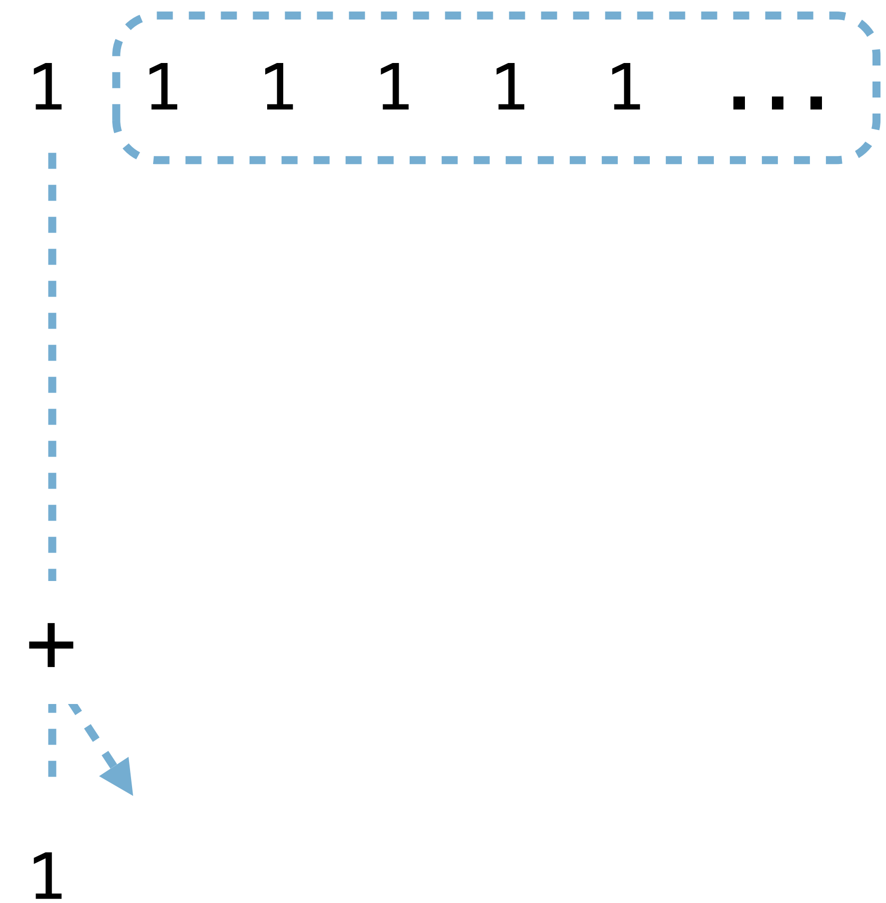

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    (add-streams ones ints)))
```

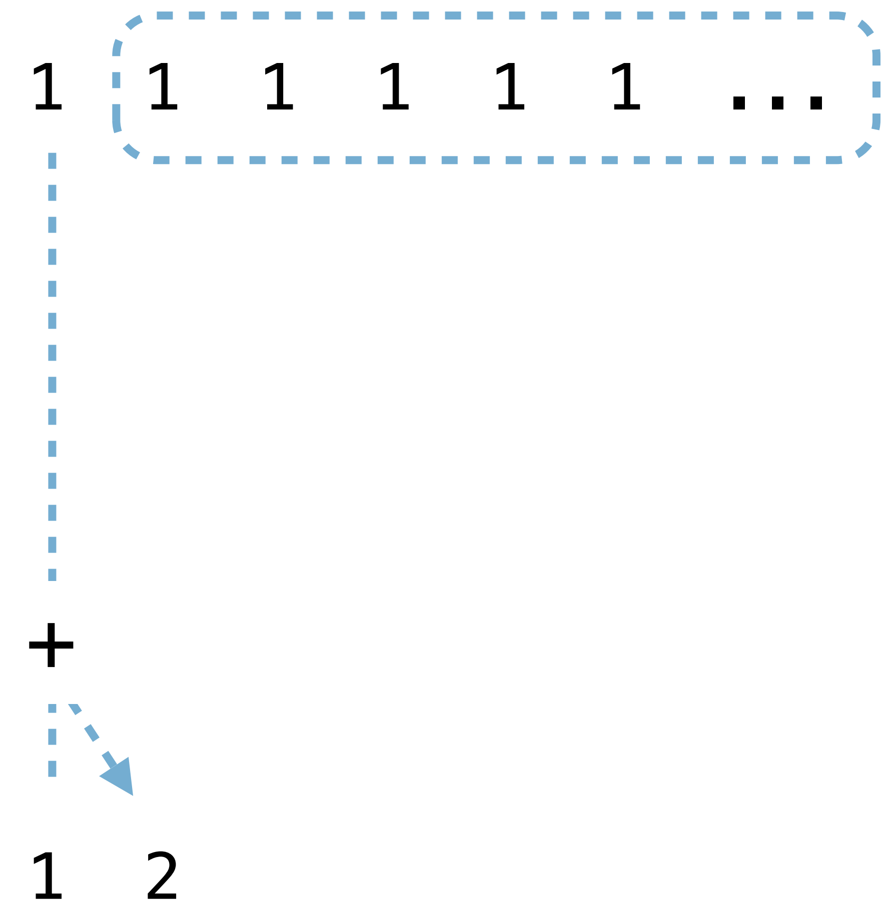

Recursively Defined Streams

```scheme
(define ones (cons-stream 1 ones))

(define (add-streams s1 s2)
  (cons-stream
    (+ (car s1) (car s2))
    (add-streams
      (cdr-stream s1)
      (cdr-stream s2))))

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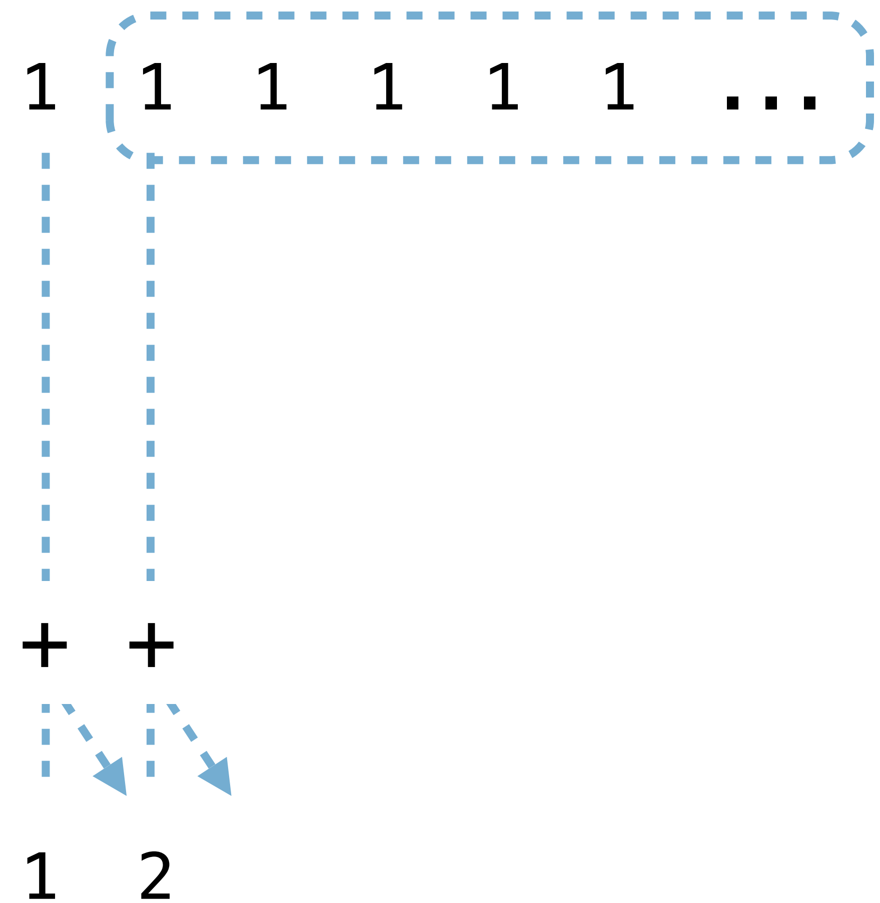

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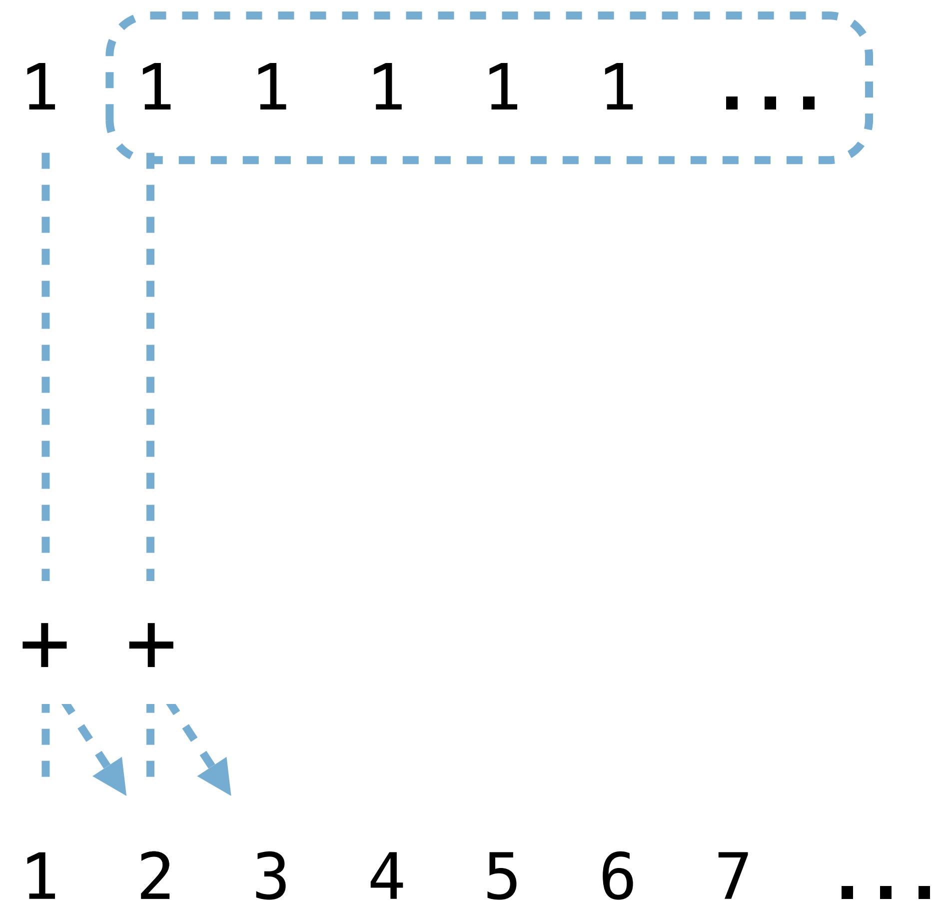

Recursively Defined Streams (demo)

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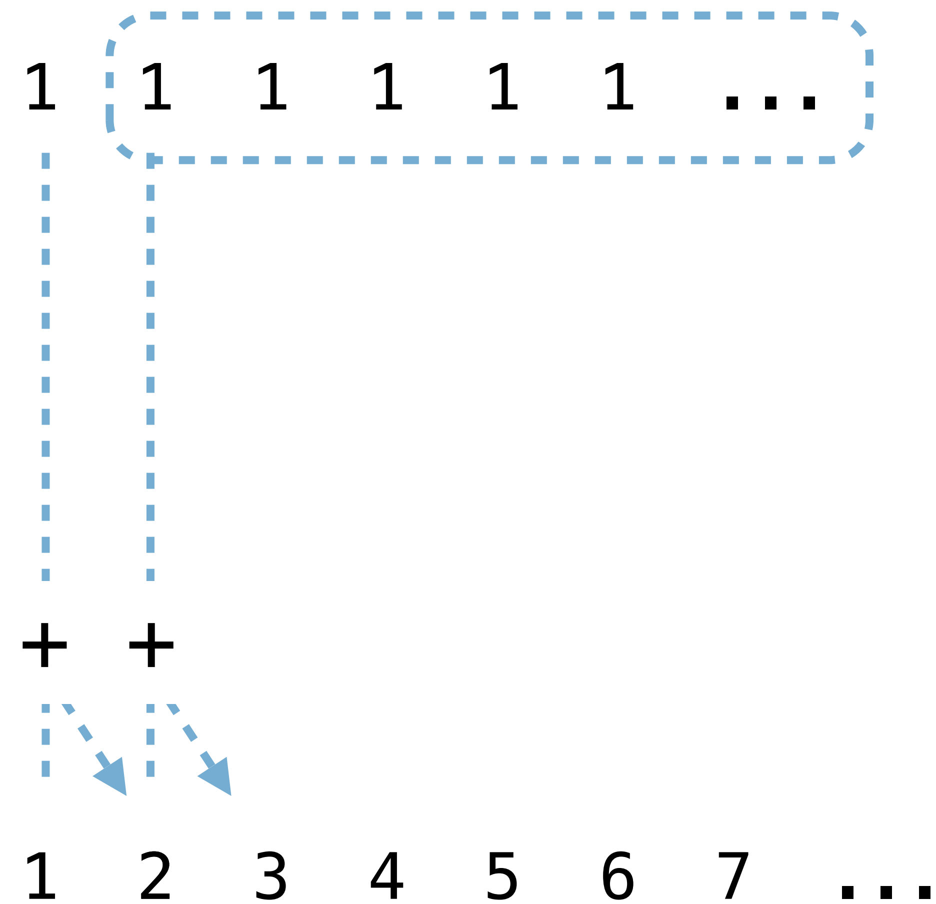

A Stream of Primes

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Break!

Symbolic Programming

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(define (square x) (* x x))
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Symbolic Programming

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Symbolic Programming

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list

- Lists can be manipulated with **car** and **cdr**
- Lists can created and combined with **cons, list, append**
- We can rewrite Scheme procedures using these tools!

List Comprehensions in Scheme

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((* x x) for x in '(1 2 3 4) if (> x 2))
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(car exp)                                       (* x x)
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                                                     x
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                                                   '(1 2 3 4)
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exp                          ((* x x) for x in '(1 2 3 4) if (> x 2))
(car exp)                                                     (* x x)
(car (cddr exp))                                                    x
(car (cddr (cddr exp)))                                   '(1 2 3 4)
(car (cddr (cddr (cddr exp))))                                (> x 2)
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`exp`	`((* x x) for x in '(1 2 3 4) if (> x 2))`
`(car exp)`	`(* x x)`
`(car (cddr exp))`	`x`
`(car (cddr (cddr exp)))`	`'(1 2 3 4)`
`(car (cddr (cddr (cddr exp))))`	`(> x 2)`

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```

```
(list 'lambda (list 'x) '(* x x))
                                    (lambda (x) (* x x))
```

```
(map (lambda (x) (* x x))
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exp                          ((* x x) for x in '(1 2 3 4) if (> x 2))
(car exp)                                                     (* x x)
(car (cddr exp))                                                    x
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(list 'lambda (list 'x) '(* x x))
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List Comprehensions in Scheme (demo)

```
exp                      ((* x x) for x in '(1 2 3 4) if (> x 2))
(car exp)                                                 (* x x)
(car (cddr exp))                                                x
(car (cddr (cddr exp)))                                '(1 2 3 4)
(car (cddr (cddr (cddr exp))))                            (> x 2)
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(list 'lambda (list 'x) '(* x x))
                                  (lambda (x) (* x x))
(list 'lambda (list 'x) '(> x 2))
                                  (lambda (x) (> x 2))
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   (map (lambda (x) (* x x))
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More Symbolic Programming

Rational numbers!

Summary

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Summary

- Tail call optimization allows some recursive procedures to take up a constant amount of space — just like iterative functions in Python!
- Streams can be used to define implicit sequences
- We can manipulate Scheme programs (as lists) to create new Scheme programs
  - This is one huge language feature that has contributed to Lisp's staying power over the years
  - Look up "macros" to learn more!