Lecture 18: Mutable Trees

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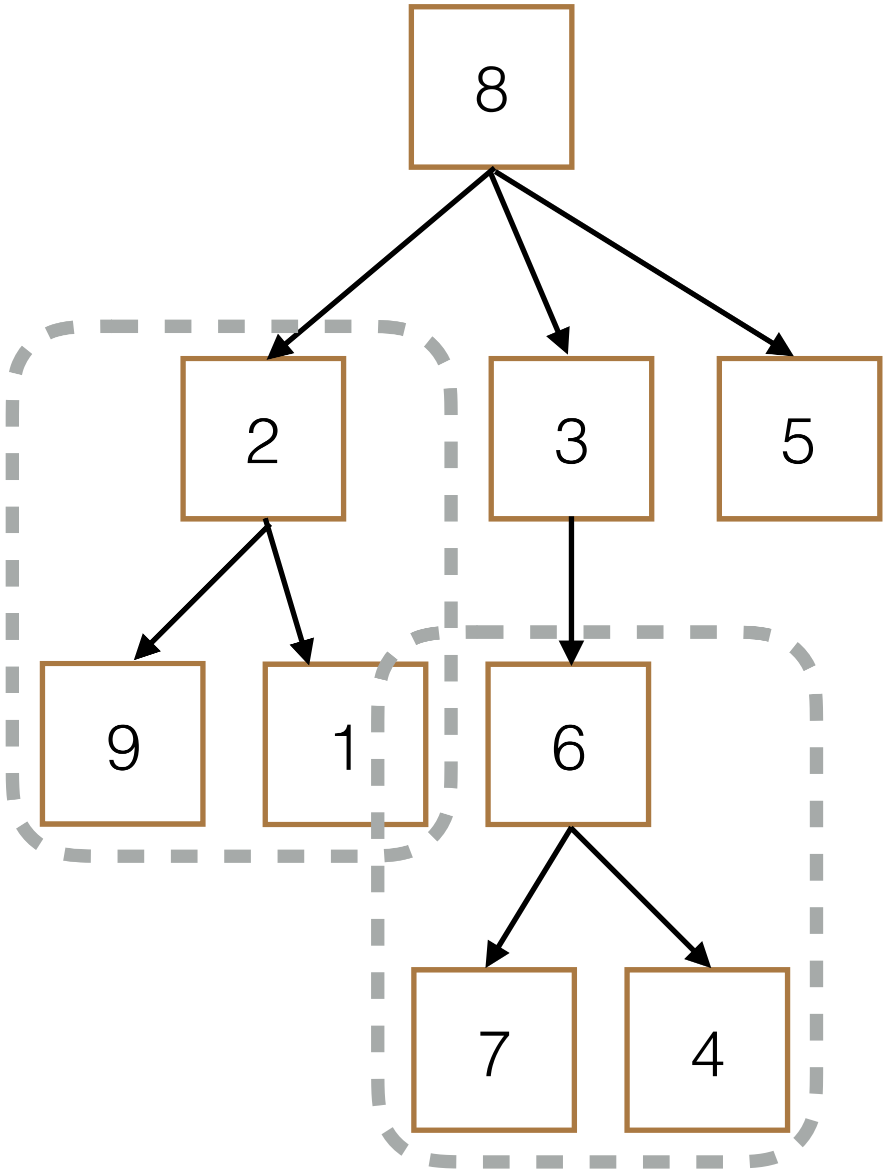
Announcements

Trees

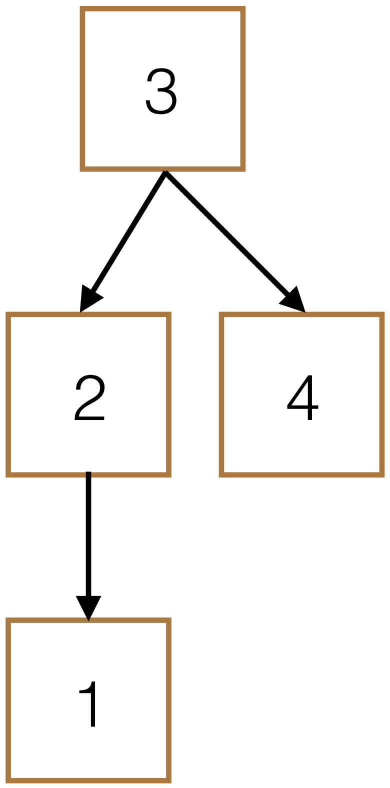
Terminology

- **Node**: single unit containing an entry
- **Root**: top node
- **Leaf**: a node with no children
- **Children**: subtree with a parent



Tree Class

```
class Tree:
    def __init__(self, entry, children=[]):
        for c in children:
            assert isinstance(c, Tree)
        self.entry = entry
        self.children = children

    def is_leaf(self):
        return not self.children

>>> t = Tree(3, [Tree(2, [Tree(1)]), Tree(4)])
>>> t.entry
3
>>> t.children[0].entry
2
>>> t.children[1].is_leaf()
True
```

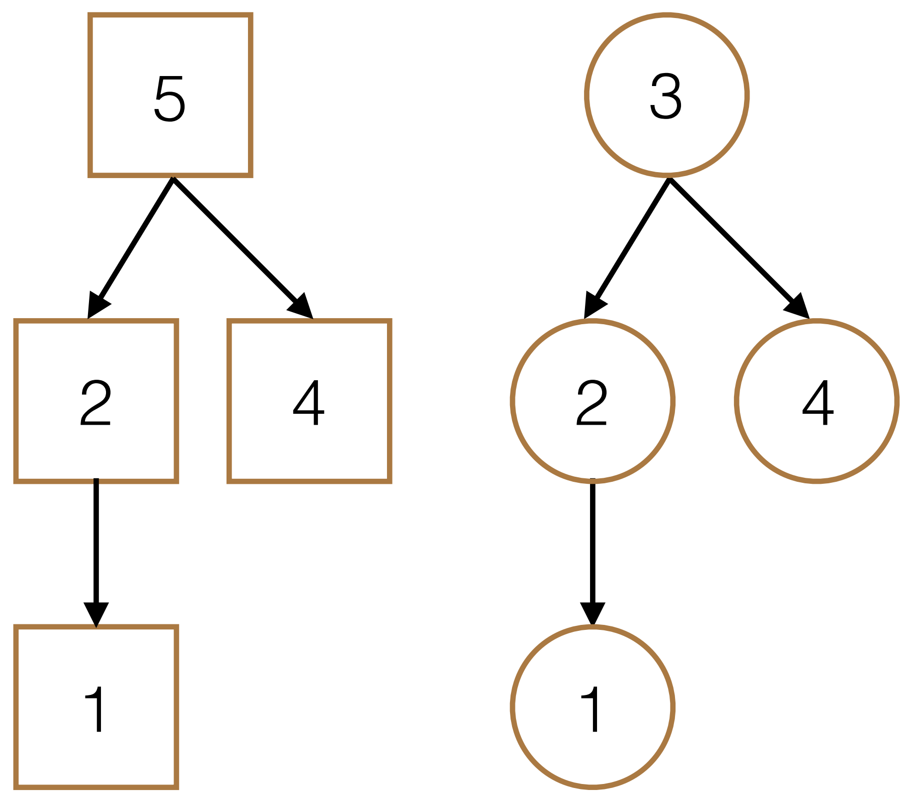


Comparison to ADT

```
class Tree:
    def __init__(self, entry,
                        children=[]):
        for c in children:
            assert isinstance(c, Tree)
        self.entry = entry
        self.children = children
```

```
def tree(entry, children=[]):
    return [entry, children]

def entry(tree):
    return tree[0]

def children(tree):
    return tree[1]
```

```
>>> t_class = Tree(3, [Tree(2,
...     [Tree(1)]), Tree(4)])
>>> t_adt = tree(3, [tree(2,
...     [tree(1)]), tree(4)])
>>> t_class.entry == entry(t_adt)
True
>>> t_class.entry = 5
>>> entry(t_adt) = 5
SyntaxError: can't assign ...
>>> t_class.entry == entry(t_adt)
False
```



Map

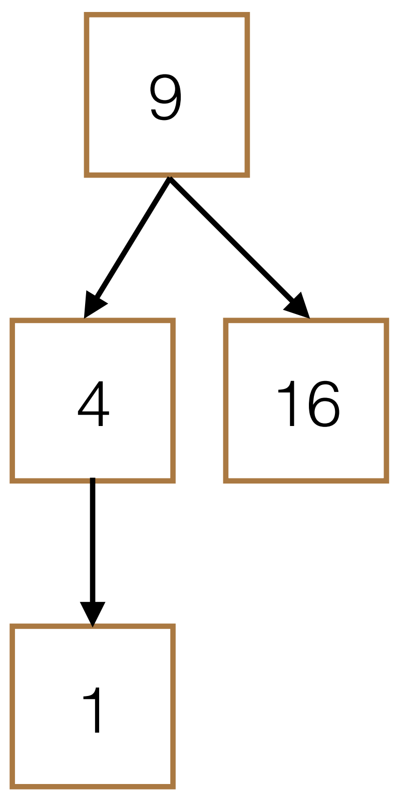
- Want to apply a function `fn` to each element in the tree

- Main Ideas
  - Apply `fn` to current node (mutate tree)
  - Call `map` on children

```
class Tree:
    def __init__(self, entry,
              children=[]): ...

    def map(self, fn):
        self.entry = fn(self.entry)
        for c in self.children:
            c.map(fn)
```

Map

```
class Tree:
    def __init__(self, entry,
            children=[]): ...

    def map(self, fn):
        self.entry = fn(self.entry)
        for c in self.children:
            c.map(fn)

>>> square = lambda x: x * x
>>> t = Tree(3, [Tree(2, [Tree(1)]), Tree(4)])
>>> t.map(square)
```



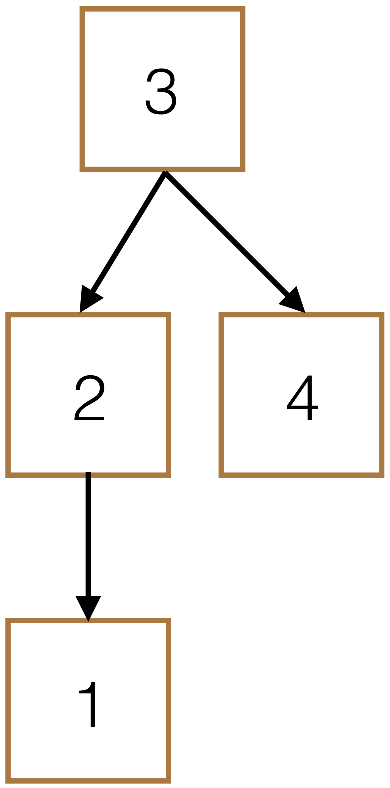
Existence

- Does the tree contain element **e**?
- Main Ideas
  - Check entry of current node
  - Otherwise, check children
    - If no children to investigate, return False

```
class Tree:
    def __init__(self, entry,
                 children=[]): ...

    def __contains__(self, e):
        if self.entry == e:
            return True
        for c in self.children:
            if e in c:
                return True
        return False
```

Existence

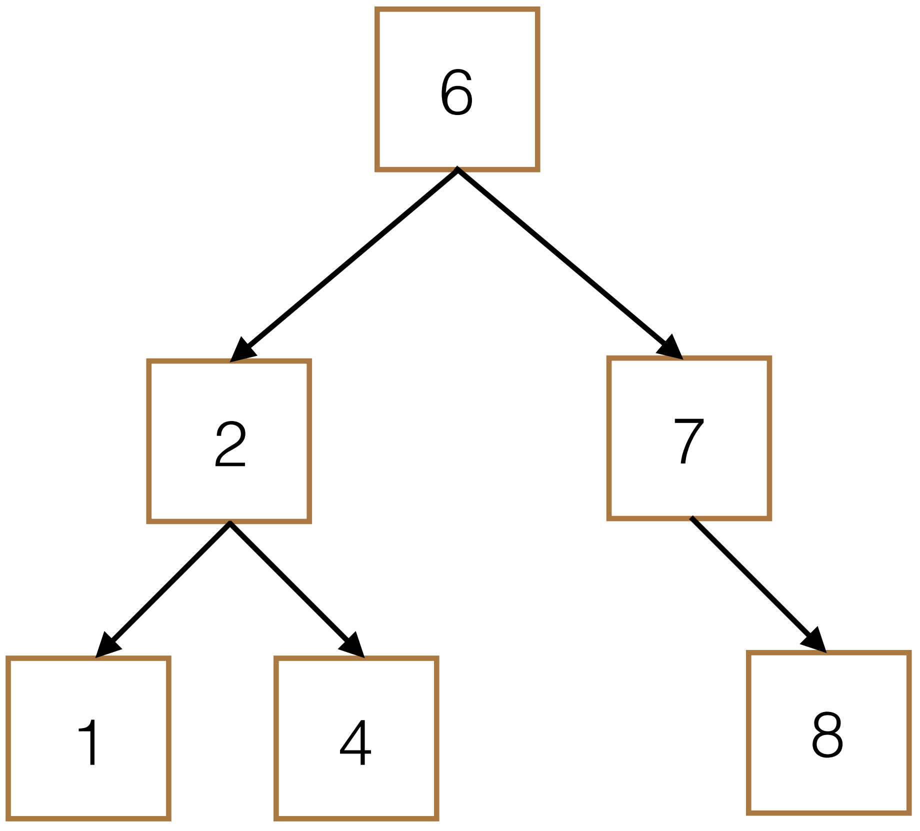
```
class Tree:
    def __init__(self, entry, children=[]): ...

    def __contains__(self, e):
        if self.entry == e:
            return True
        for c in self.children:
            if e in c:
                return True
        return False

>>> t = Tree(3, [Tree(2, [Tree(1)]), Tree(4)])
>>> 8 in t
False
>>> 2 in t
True
```



Binary Search Tree

Definition

- Each node has at most 2 children, left and right
- Left child elements are all less than or equal to entry
- Right child elements are all greater than entry
- Left child and right child are also BSTs
- Only contains numbers!



Binary Search Tree Class

```
class BST:
    empty = ()
    def __init__(self, entry, left=empty, right=empty):
        assert left is BST.empty or isinstance(left, BST)
        assert right is BST.empty or isinstance(right, BST)

        self.entry = entry
        self.left, self.right = left, right

        if left is not BST.empty:
            assert left.max <= entry
        if right is not BST.empty:
            assert entry < right.min

    @property
    def max(self): ...  # Returns the maximum element in the BST

    @property
    def min(self): ...  # Returns the minimum element in the BST
```

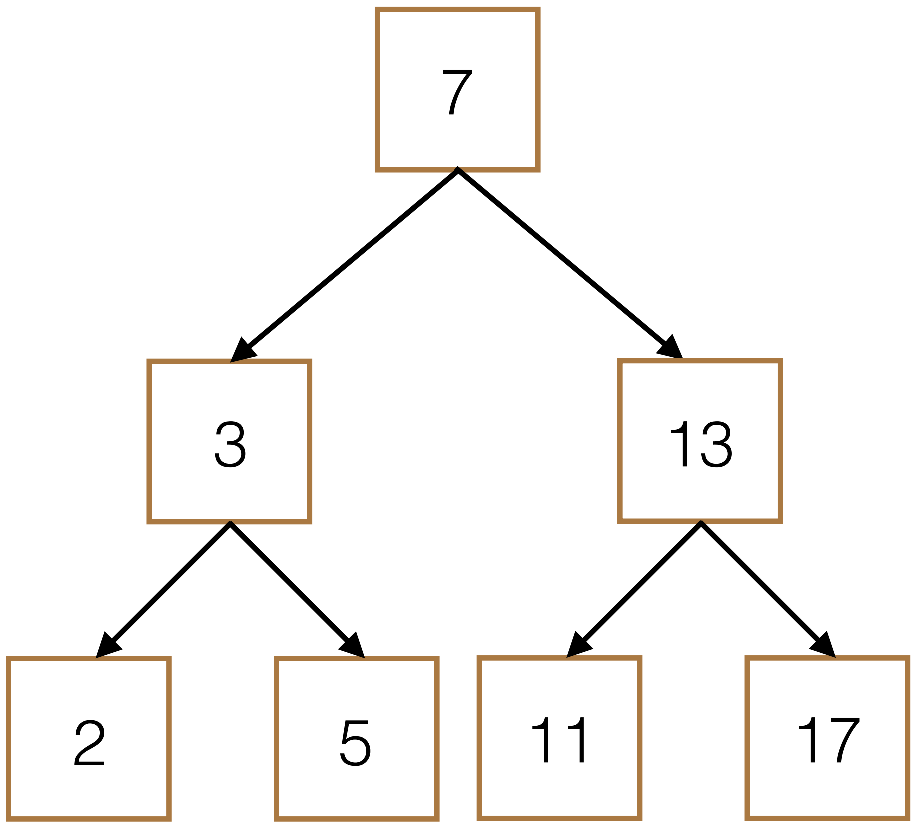
Existence

- Does the BST contain element **e**?
- Main Ideas
  - Check entry of current node
  - Otherwise, check left or right
    - If no children to investigate, return False

```
class BST:
    def __init__(self, entry,
        left=empty, right=empty): ...

    def __contains__(self, e):
        if self.entry == e:
            return True
        elif e < self.entry and self.left
                        is not BST.empty:
            return e in self.left
        elif e > self.entry and self.right
                        is not BST.empty:
            return e in self.right
        return False
```

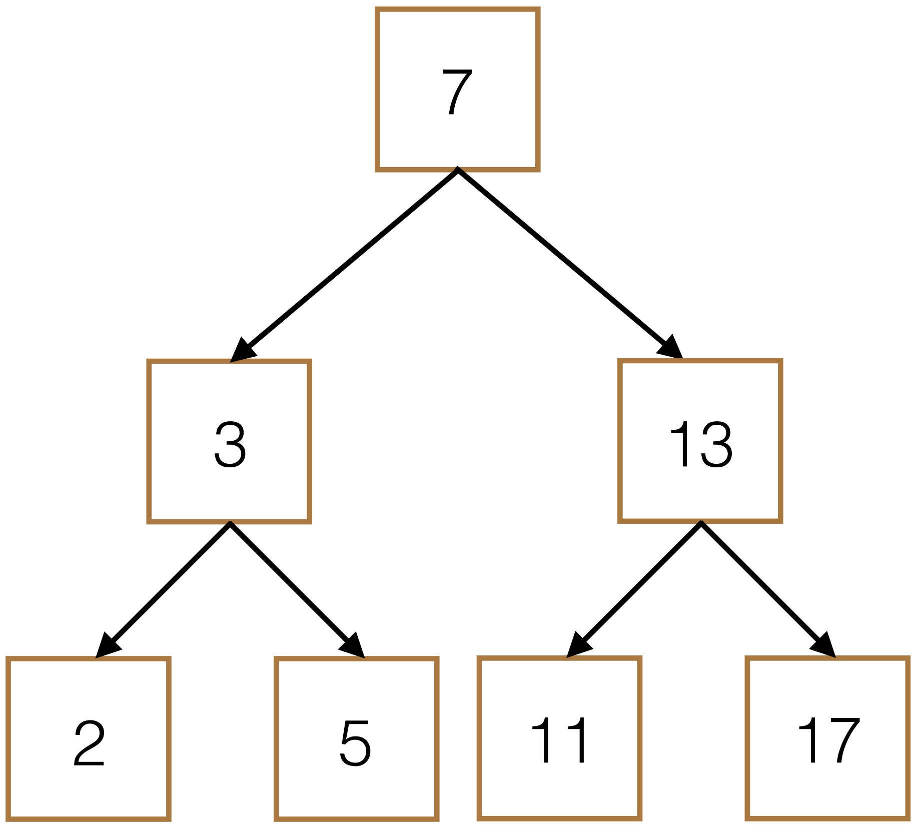
Runtime Comparison

- Is there a difference in runtime when we check existence in a tree versus a BST?
- Runtime in terms of **n**, the number of nodes



Runtime Comparison

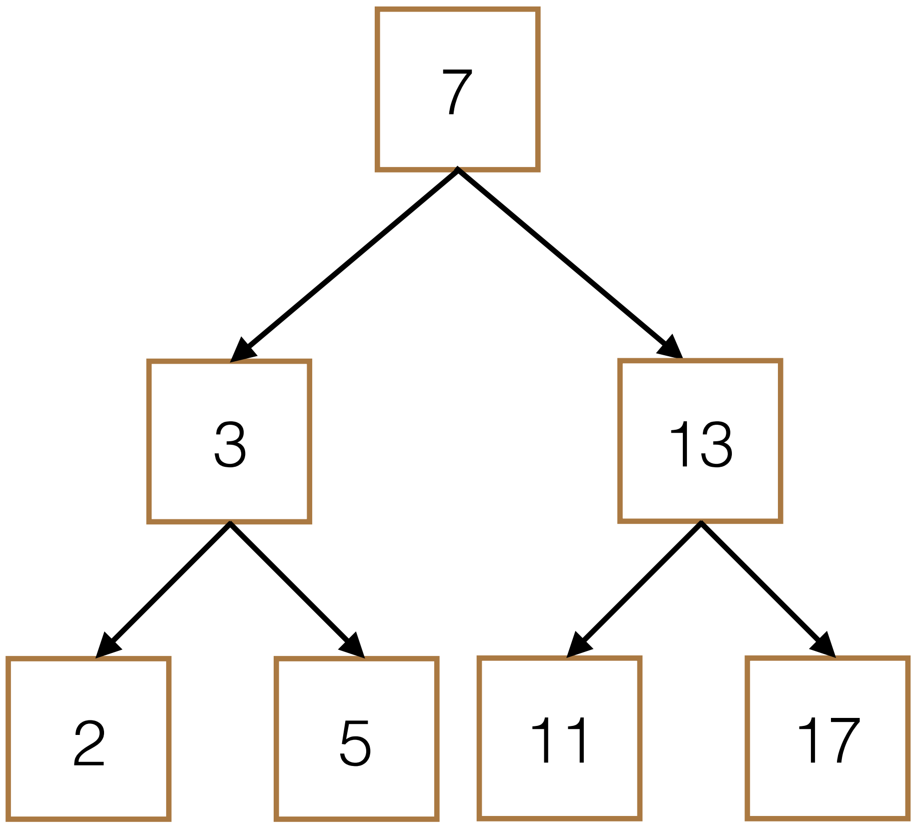
```
class Tree:
    def __init__(self, entry, children=[]): ...

    def __contains__(self, e):
        if self.entry == e:
            return True
        for c in self.children:
            if e in c:
                return True
        return False

>>> t = Tree(7, [Tree(3, [Tree(2),
...     Tree(5)]), Tree(13,
...     [Tree(11), Tree(17)])])
>>> 11 in t
True
```



$\Theta(n)$

Runtime Comparison

```
class BST:
    def __init__(self, entry, left=empty, right=empty): ...

    def __contains__(self, e):
        if self.entry == e:
            return True
        elif e < self.entry and self.left
                        is not BST.empty:
            return e in self.left
        elif e > self.entry and self.right
                        is not BST.empty:
            return e in self.right
        return False

>>> bst = BST(7,
...           BST(3, BST(2), BST(5)),
...           BST(13, BST(11), BST(17)))
>>> 11 in bst
True
```



$\Theta(\log n)$

Summary

- Trees created with a class are mutable!
- BSTs allow us to organize our data in left child and right child based on value
- BST allows for more efficient search
  - $\Theta(n)$ in regular tree
  - $\Theta(\log n)$ in BST