# Lecture 7: Tree Recursion

Brian Hou
June 29, 2016

# Announcements

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- Alternate Exam Request: goo.gl/forms/FDQix4I5dNXPQDgw2

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Ready? cs61a.org/proj/hog_contest

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# Roadmap

Introduction

Functions

Data

Mutability

Objects

Interpretation

Paradigms

Applications

- This week (Functions), the goals are:
  - To understand the idea of *functional abstraction*
  - To study this idea through:
    - higher-order functions
    - recursion
    - orders of growth

# Recursion

# The Cascade Function

# The Cascade Function (demo)

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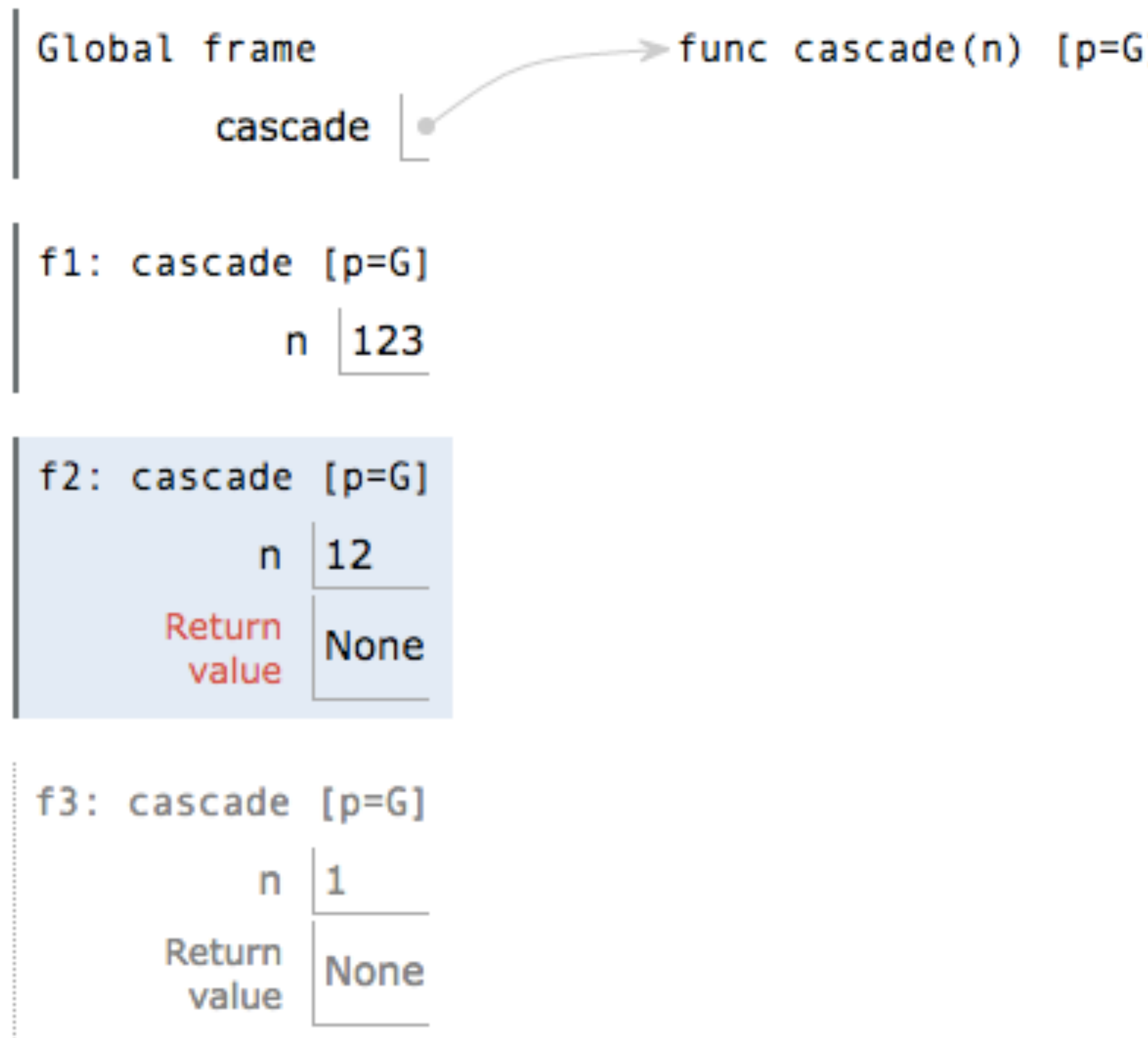
```
def cascade(n):
    if n < 10:
        print(n)
    else:
        print(n)
        cascade(n//10)
        print(n)

cascade(123)
```




**Output**

123
12
1
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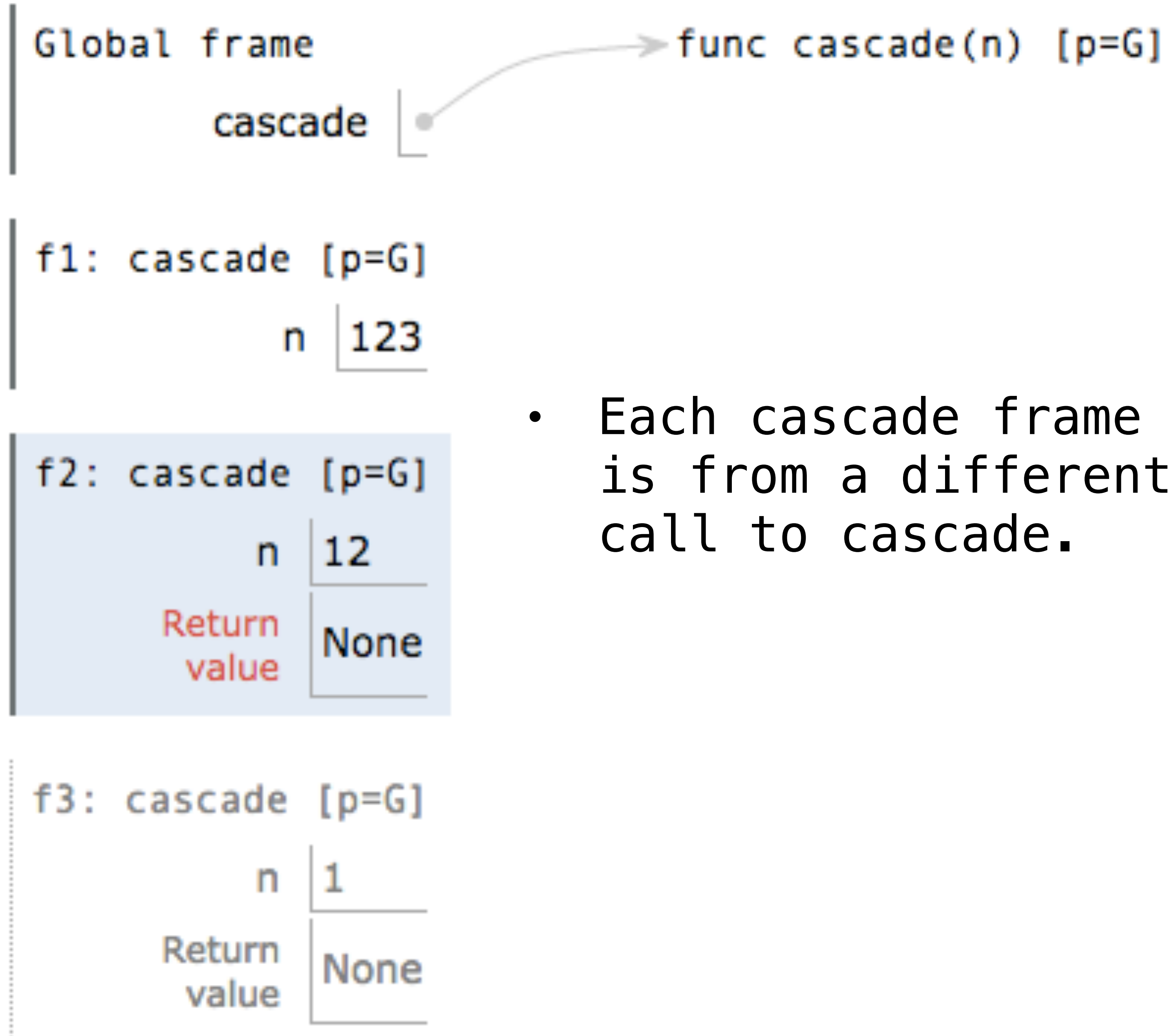


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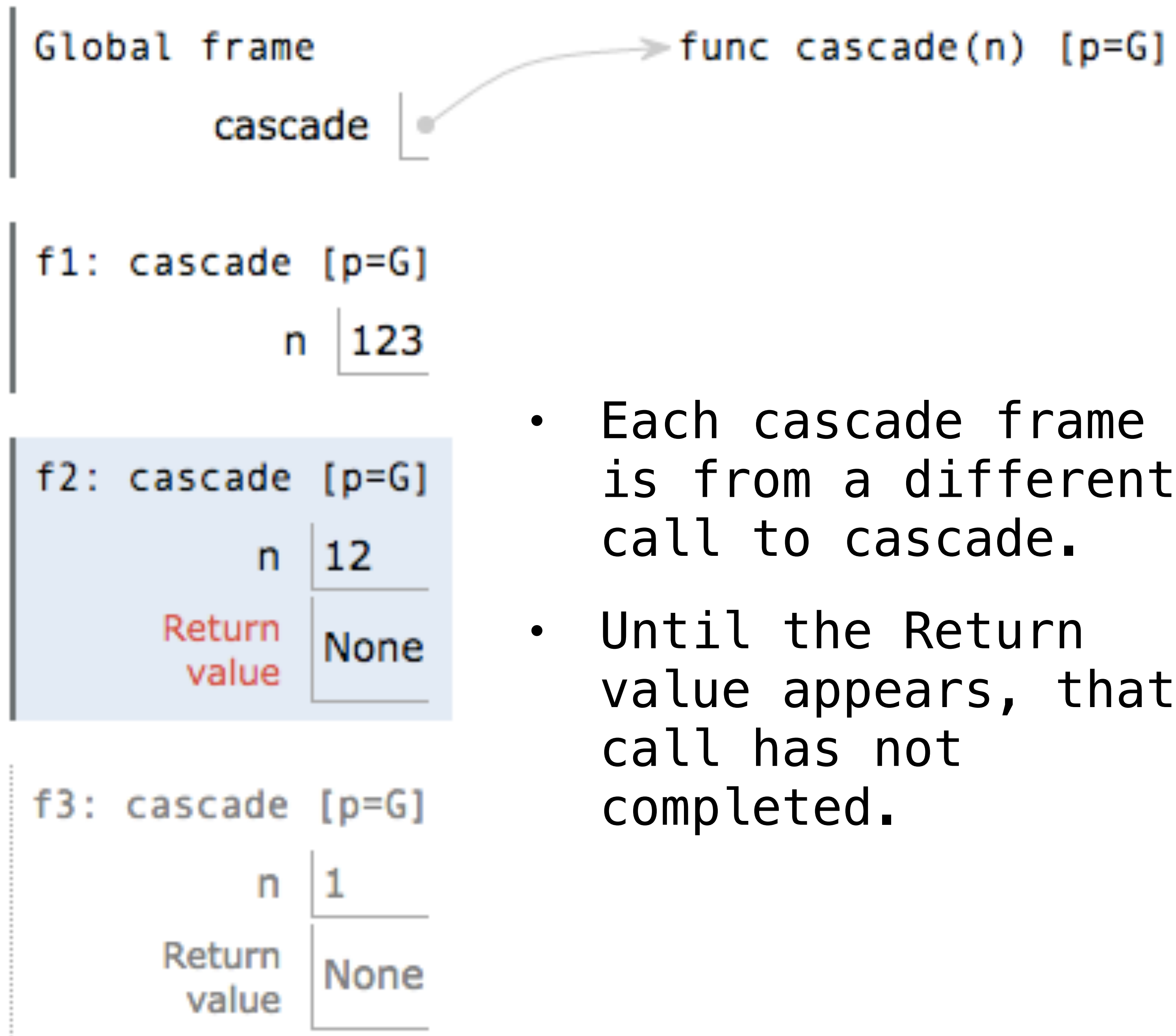


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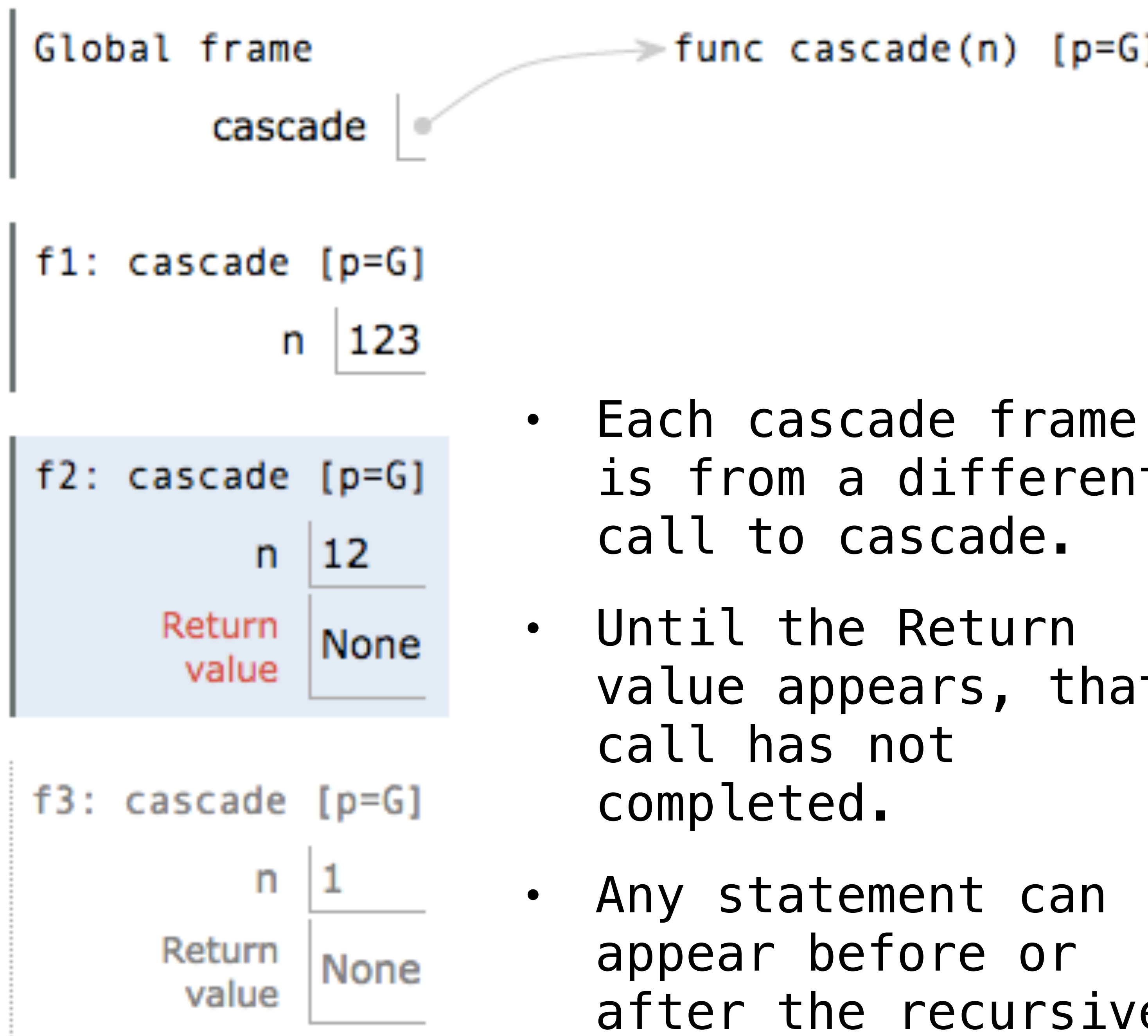


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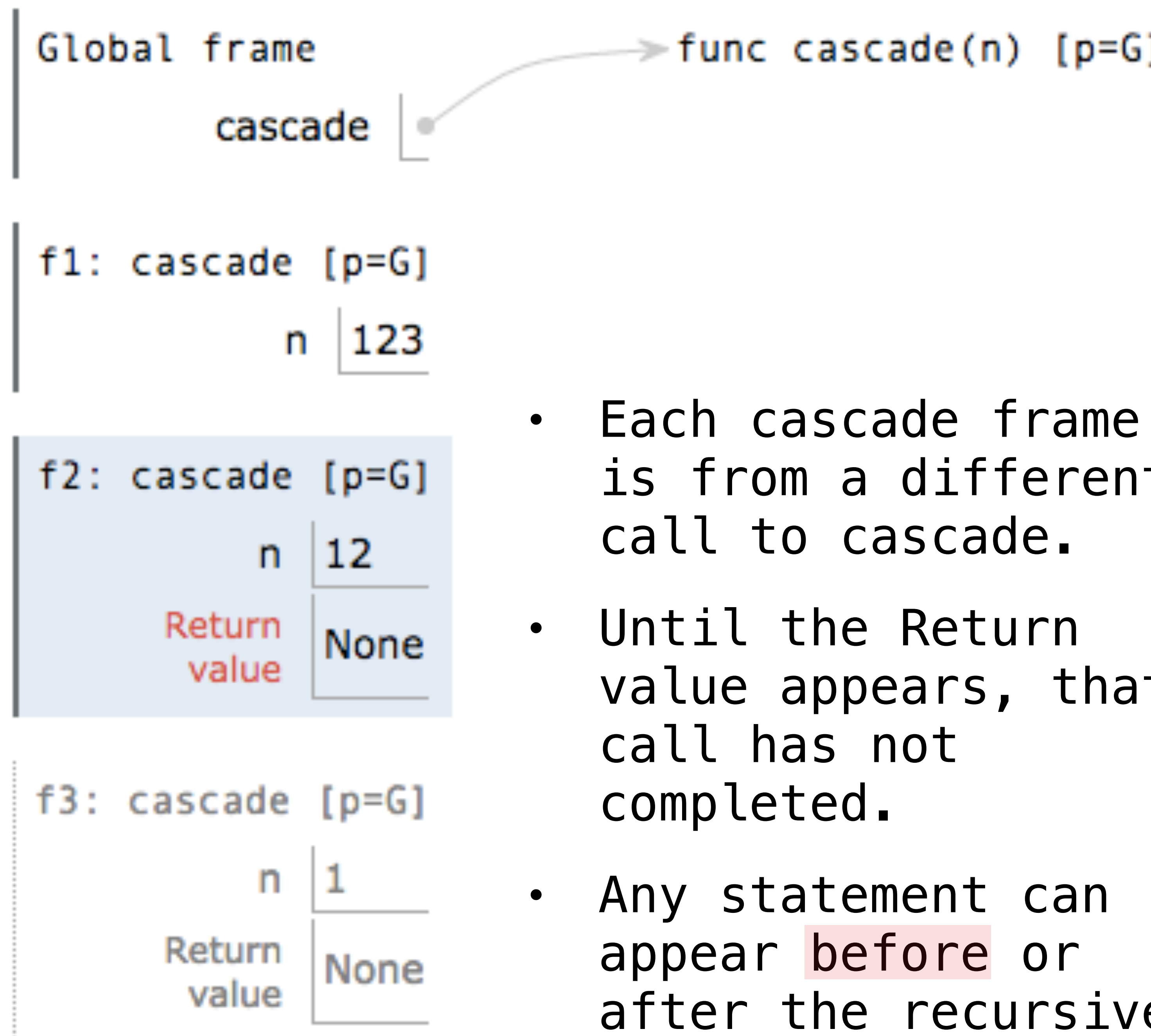


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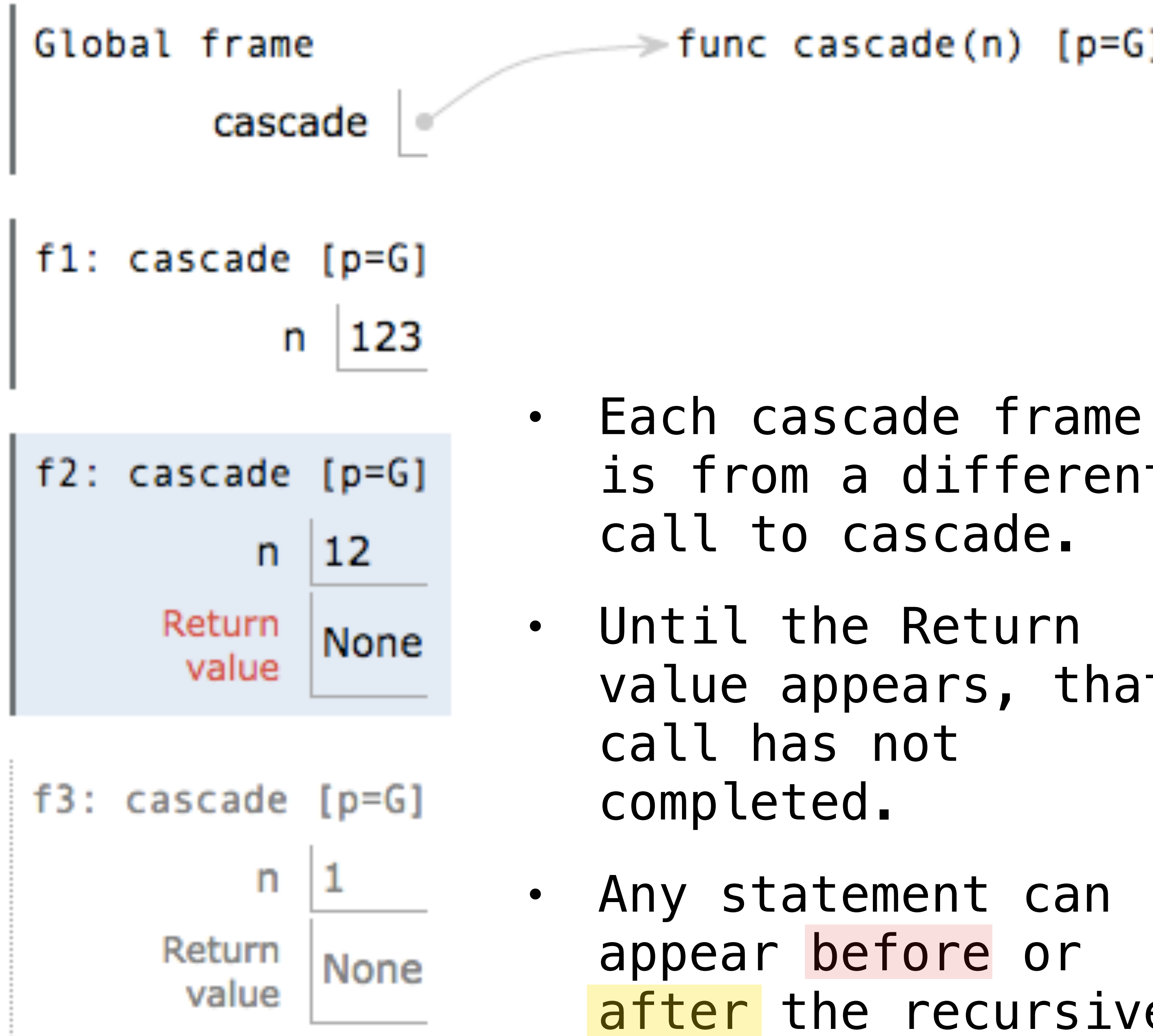


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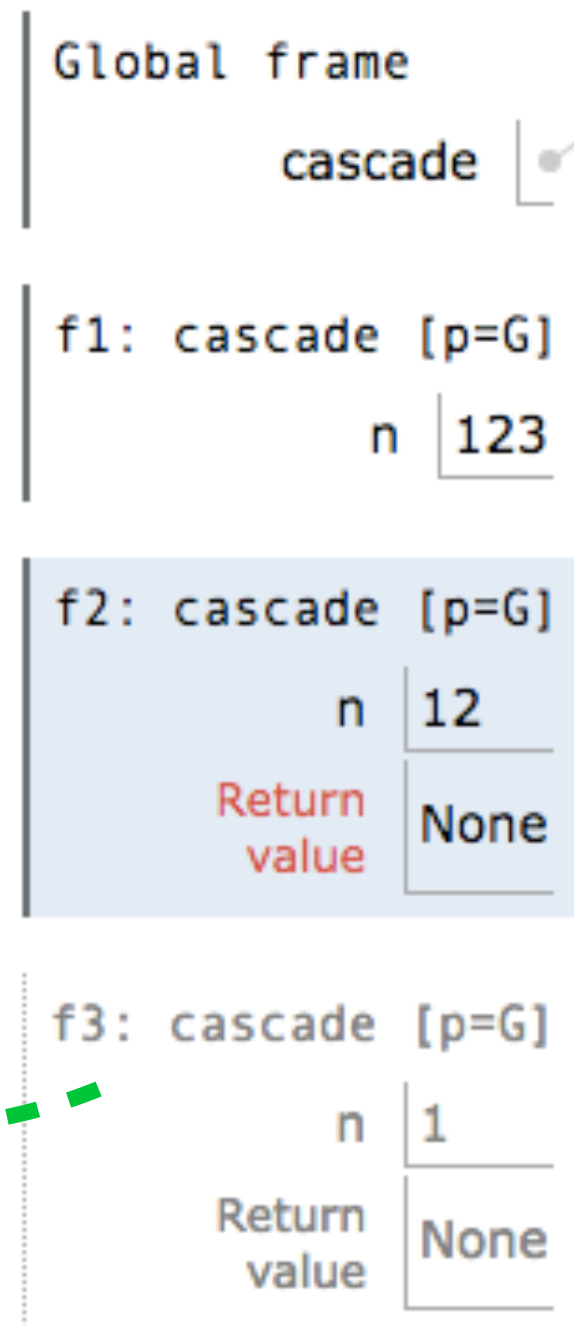


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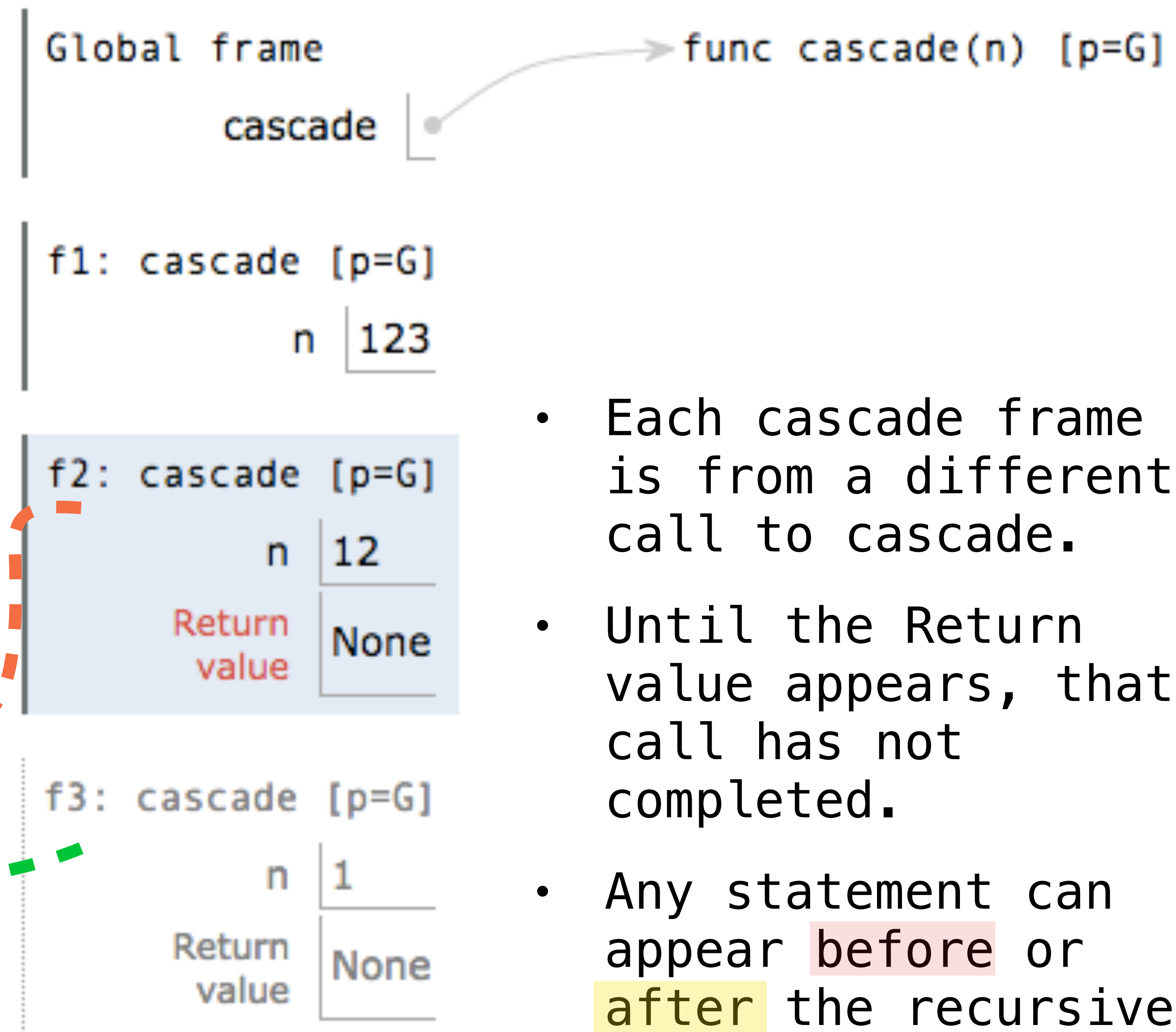


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# Two Definitions of Cascade

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```

```
def cascade(n):
    print(n)
    if n >= 10:
        cascade(n // 10)
        print(n)
```

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- If two implementations are equally clear, then shorter is usually better

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- In this case, the longer implementation is more clear (to me)

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        print(n)
```

- If two implementations are equally clear, then shorter is usually better
- In this case, the longer implementation is more clear (to me)
- When learning to write recursive functions, put base cases first

# Inverse Cascade

# Inverse Cascade

**Output**

```
1
12
123
1234
123
12
1
```

```
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```

# Inverse Cascade

**Output**

```
1
12
123
1234
123
12
1
```

```python
def inverse_cascade(n):
    grow(n)
    print(n)
    shrink(n)
```

```python
def f_then_g(f, g, n):
    if n:
        f(n)
        g(n)
```

# Inverse Cascade

**Output**

```
1
12
123
1234
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12
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```

```
def inverse_cascade(n):
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```

```
grow   = lambda n: f_then_g(                    )

shrink = lambda n: f_then_g(                    )
```

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```

```
grow   = lambda n: f_then_g(grow,  print,  n // 10)

shrink = lambda n: f_then_g(print, shrink, n // 10)
```

# Fibonacci

# The Fibonacci Sequence

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**n:** 0, 1, 2, 3, 4, 5, 6, 7, 8,

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| | |
|---|---|
| **n:** | 0, 1, 2, 3, 4, 5, 6, 7, 8, |
| **fib(n):** | 0, 1, 1, 2, 3, 5, 8, 13, 21, |

# The Fibonacci Sequence

| | |
|---|---|
| **n:** | 0, 1, 2, 3, 4, 5, 6, 7, 8, ... , 35 |
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def fib(n):
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    while k < n:
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The next Fibonacci number is the sum of the two previous Fibonacci numbers

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def fib(n):
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This correction was made on July 3 at 10PM

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def fib(n):
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    elif n == 1:
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```

# The Fibonacci Sequence

**n:** 0, 1, 2, 3, 4, 5, 6, 7, 8,

**fib(n):** 0, 1, 1, 2, 3, 5, 8, 13, 21,

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The next Fibonacci number is the sum of the two previous Fibonacci numbers

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# Tree Recursion

Tree-shaped processes arise whenever executing the body of a recursive function makes more than one recursive call

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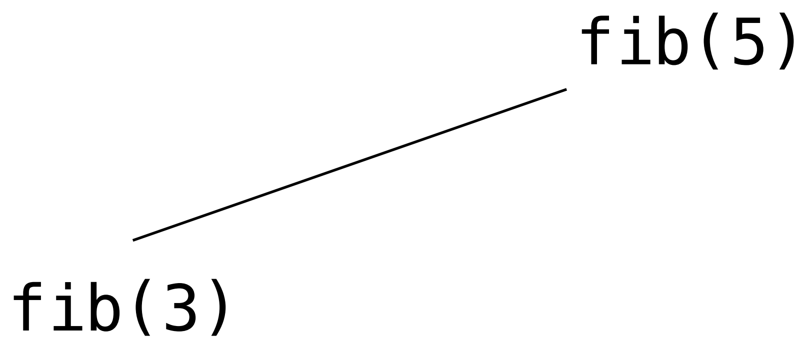
# A Tree-Recursive Process

# A Tree-Recursive Process

fib(5)

# A Tree-Recursive Process



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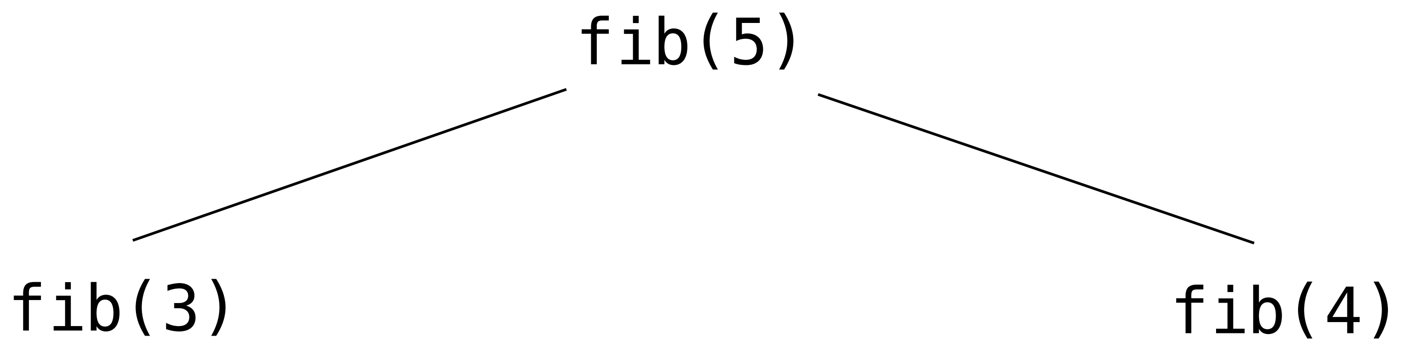

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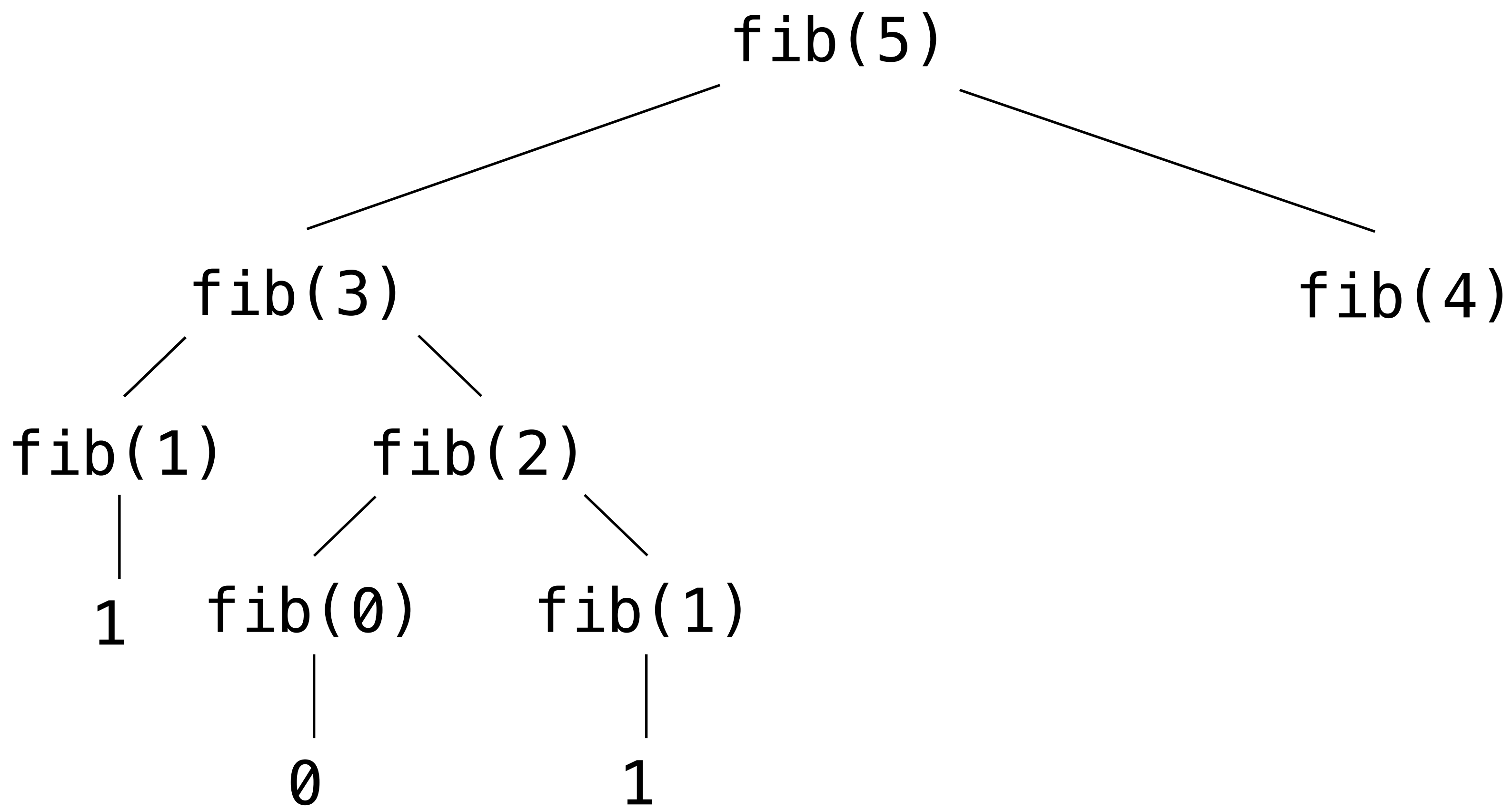

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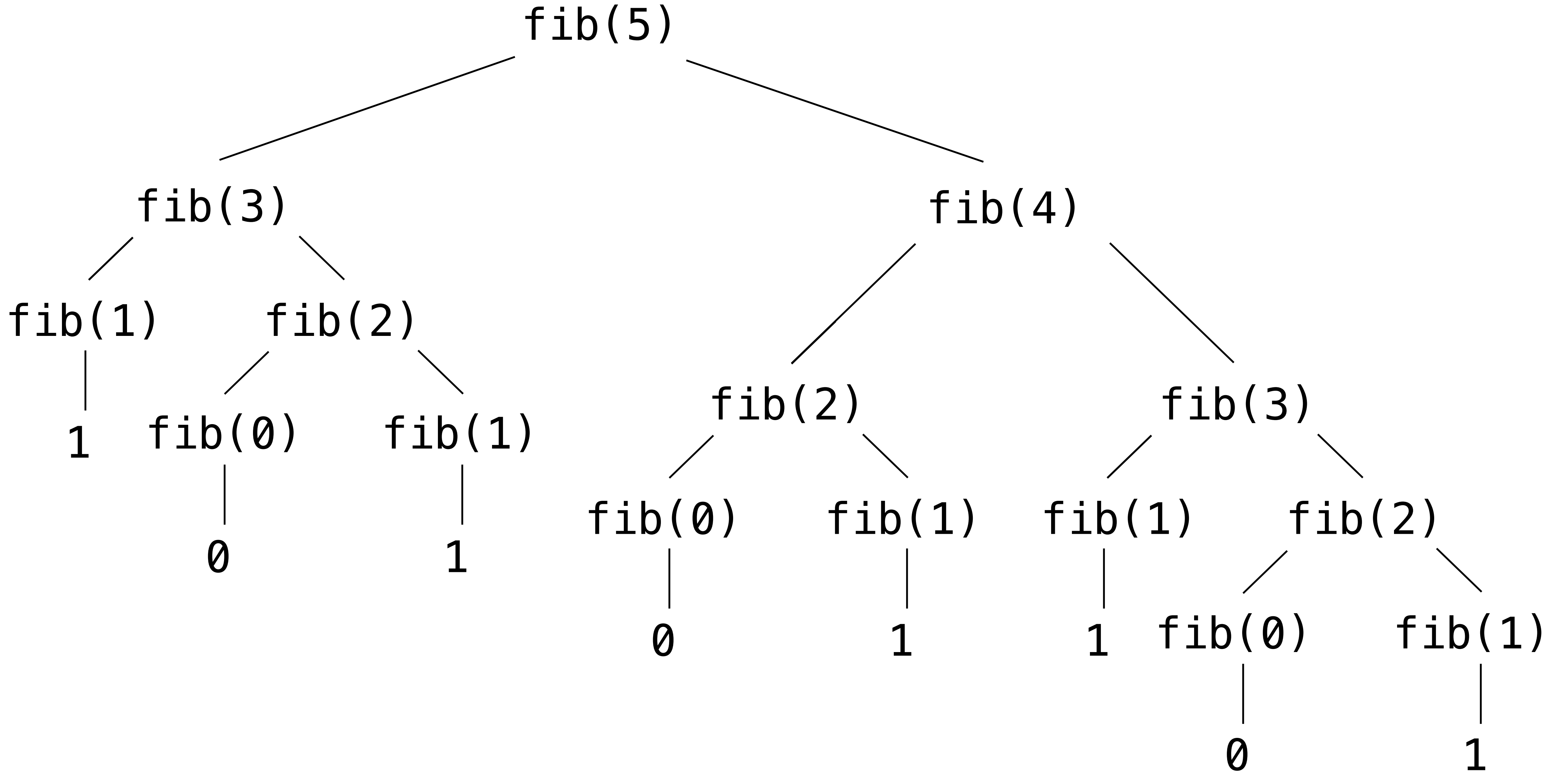

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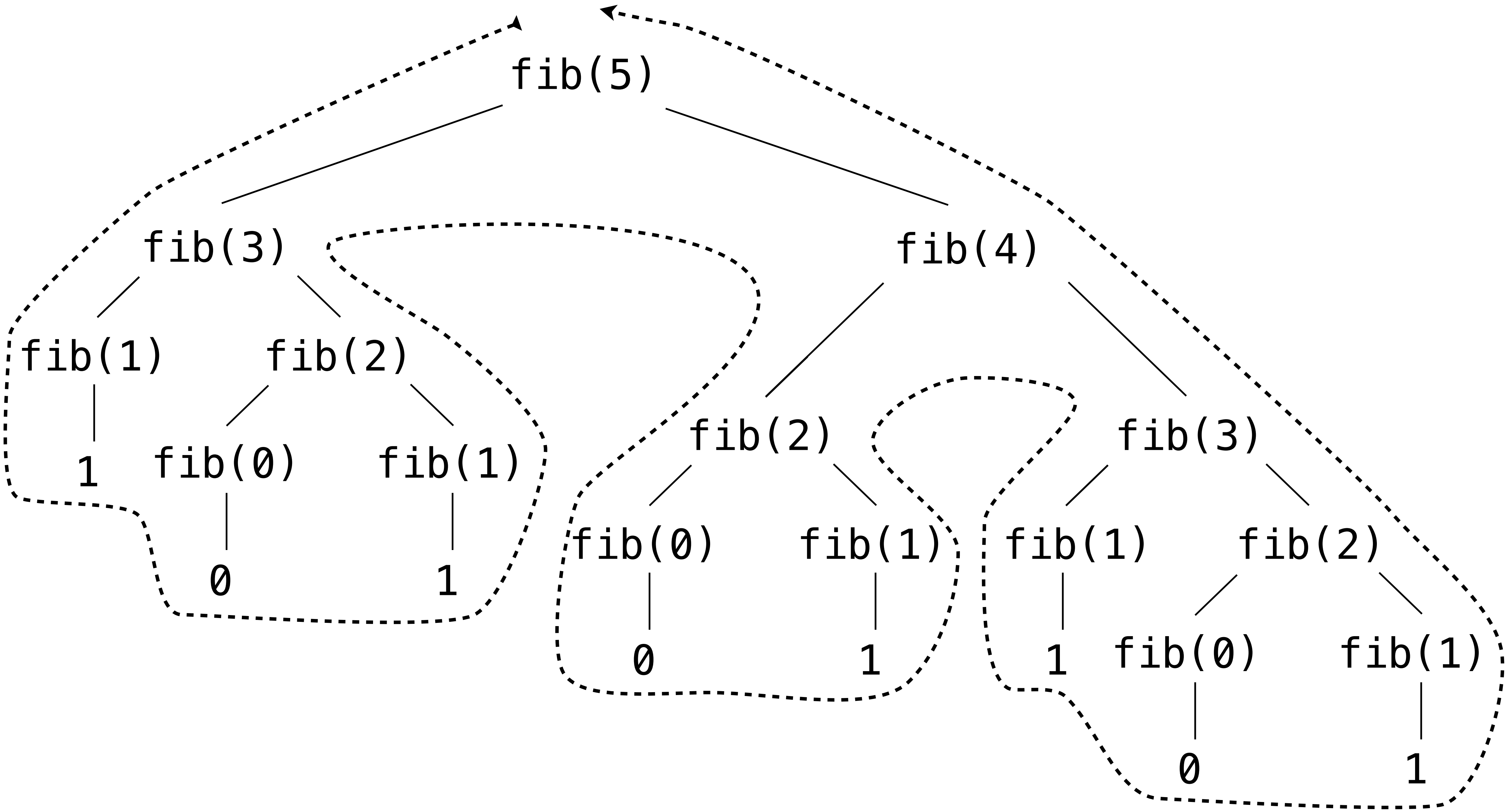

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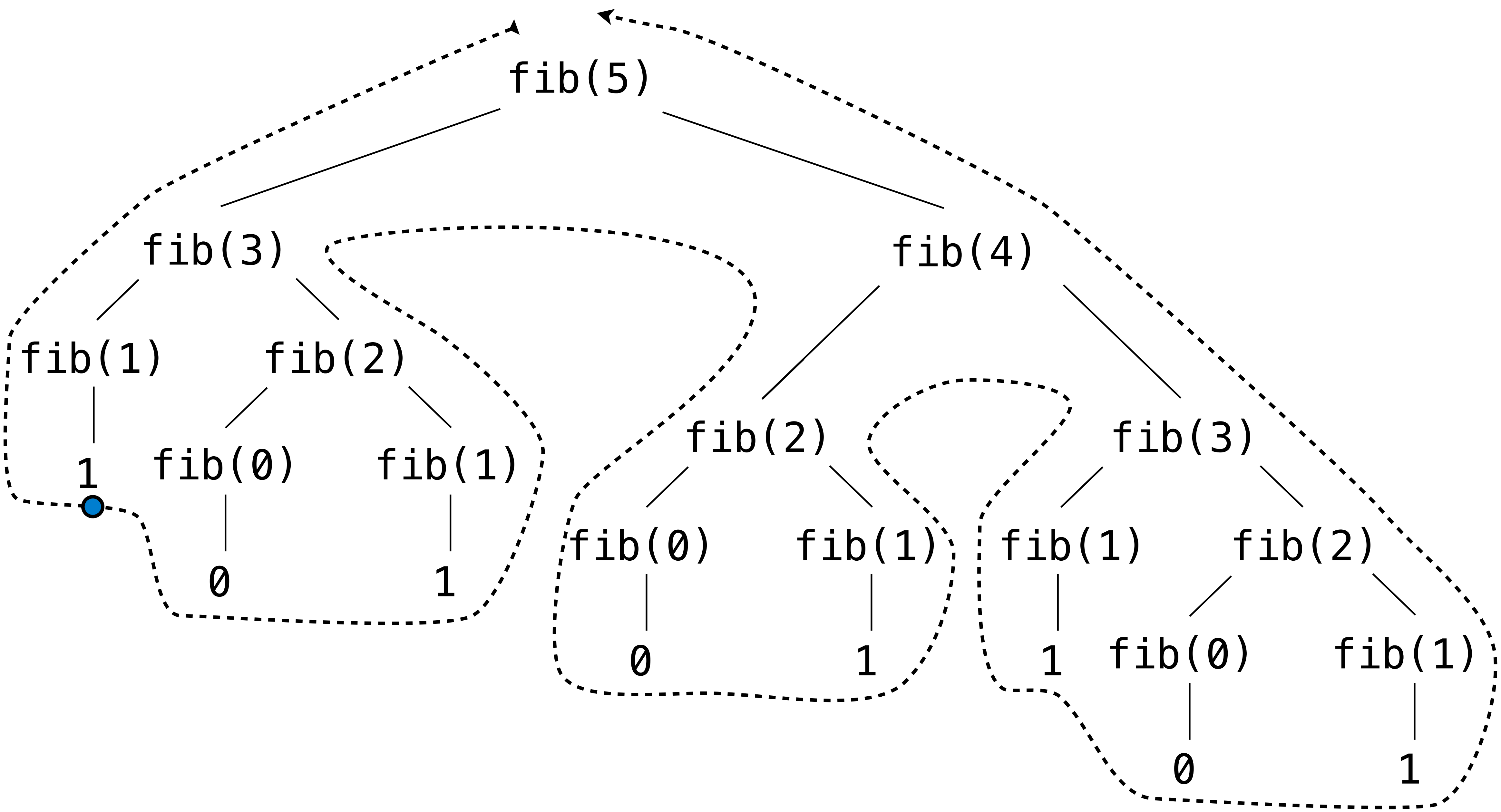

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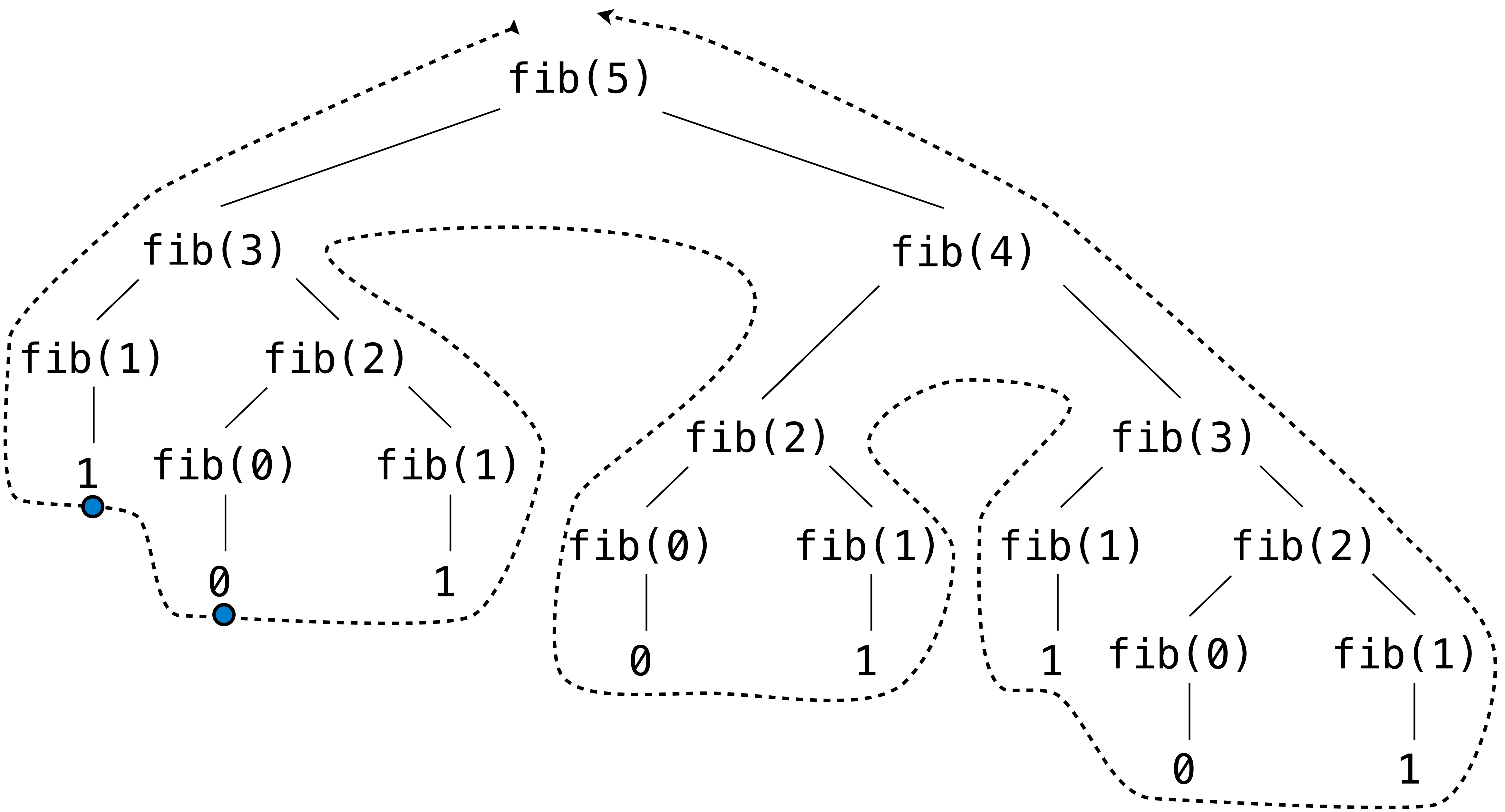

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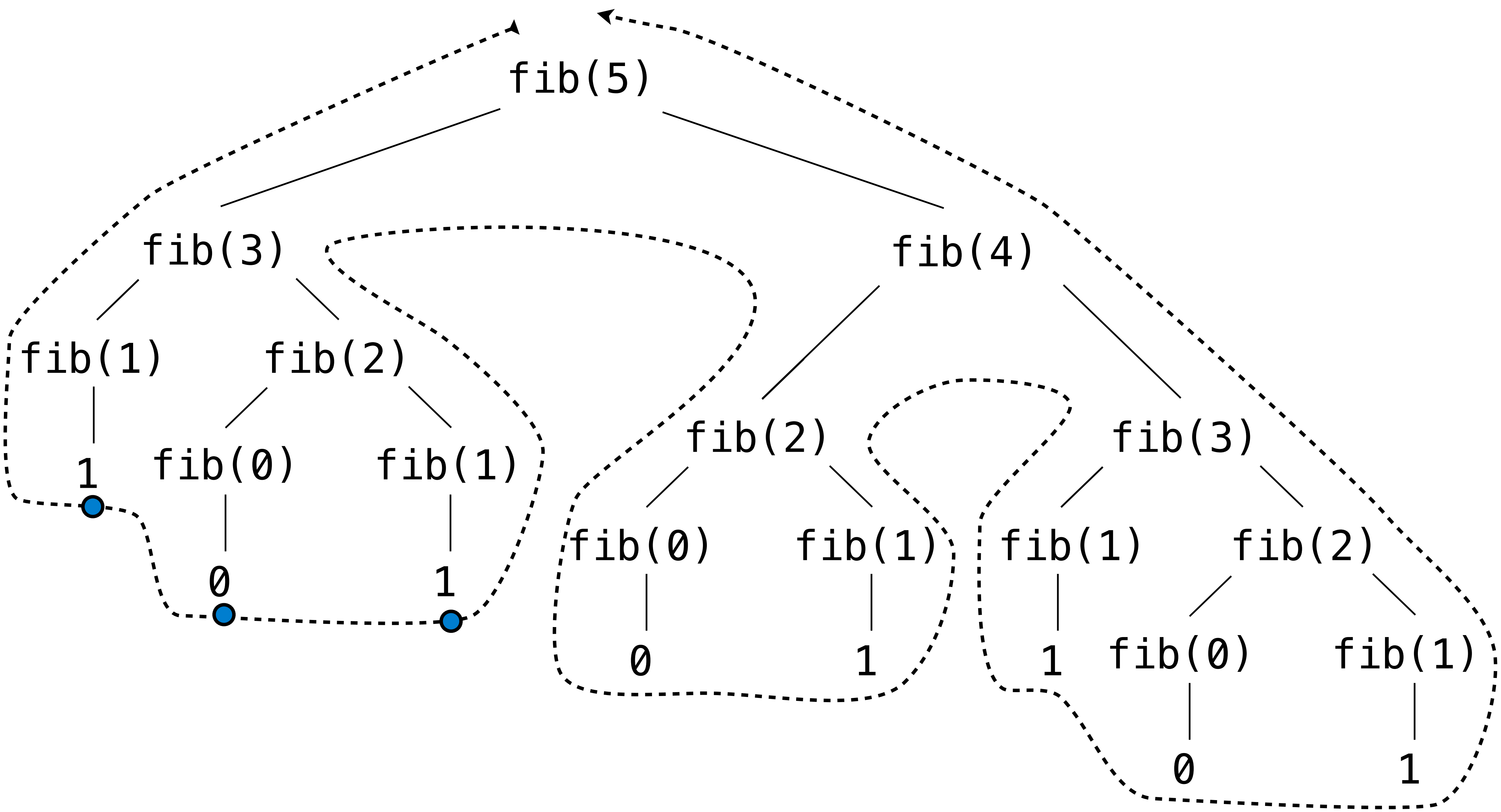

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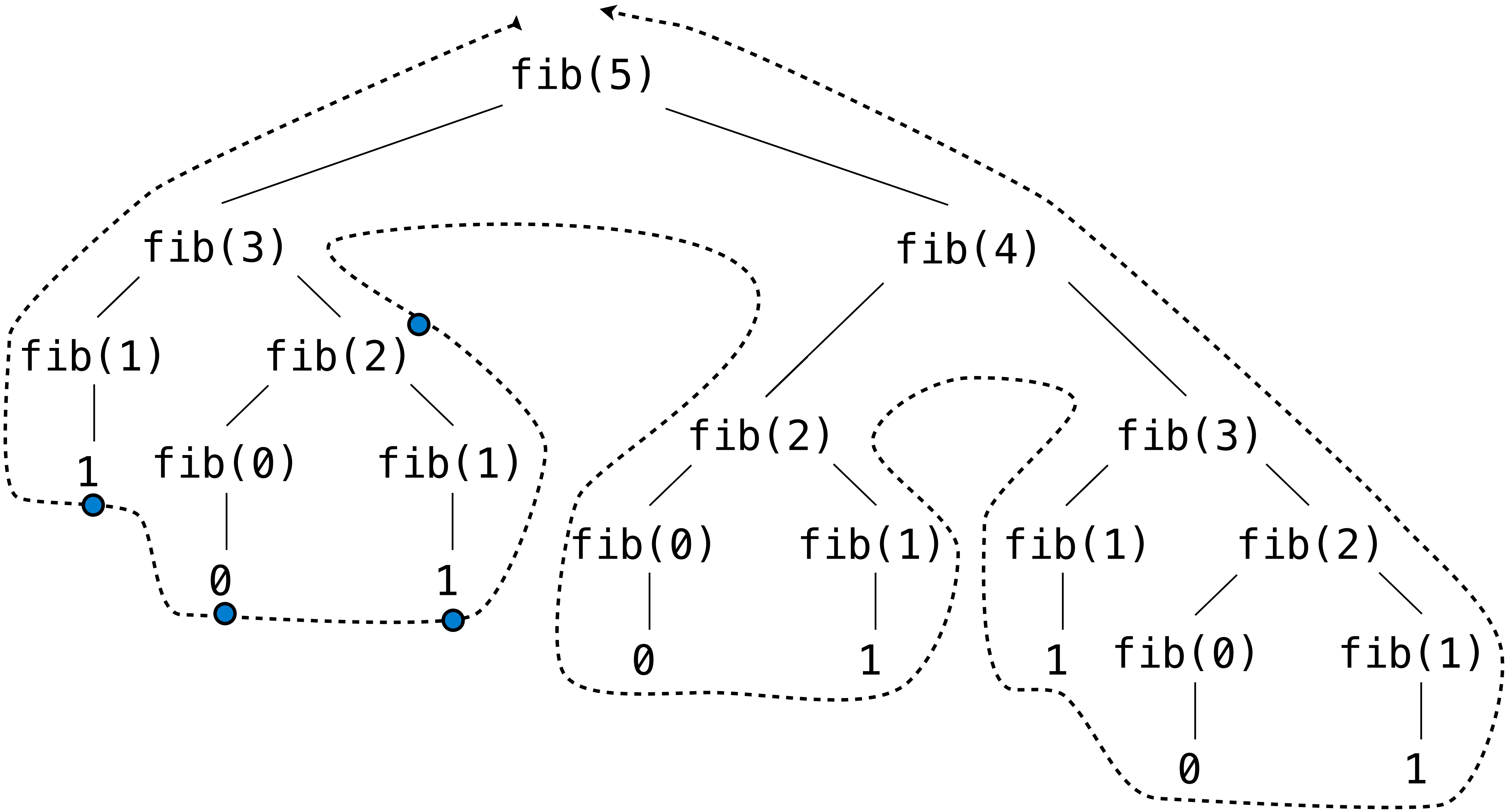

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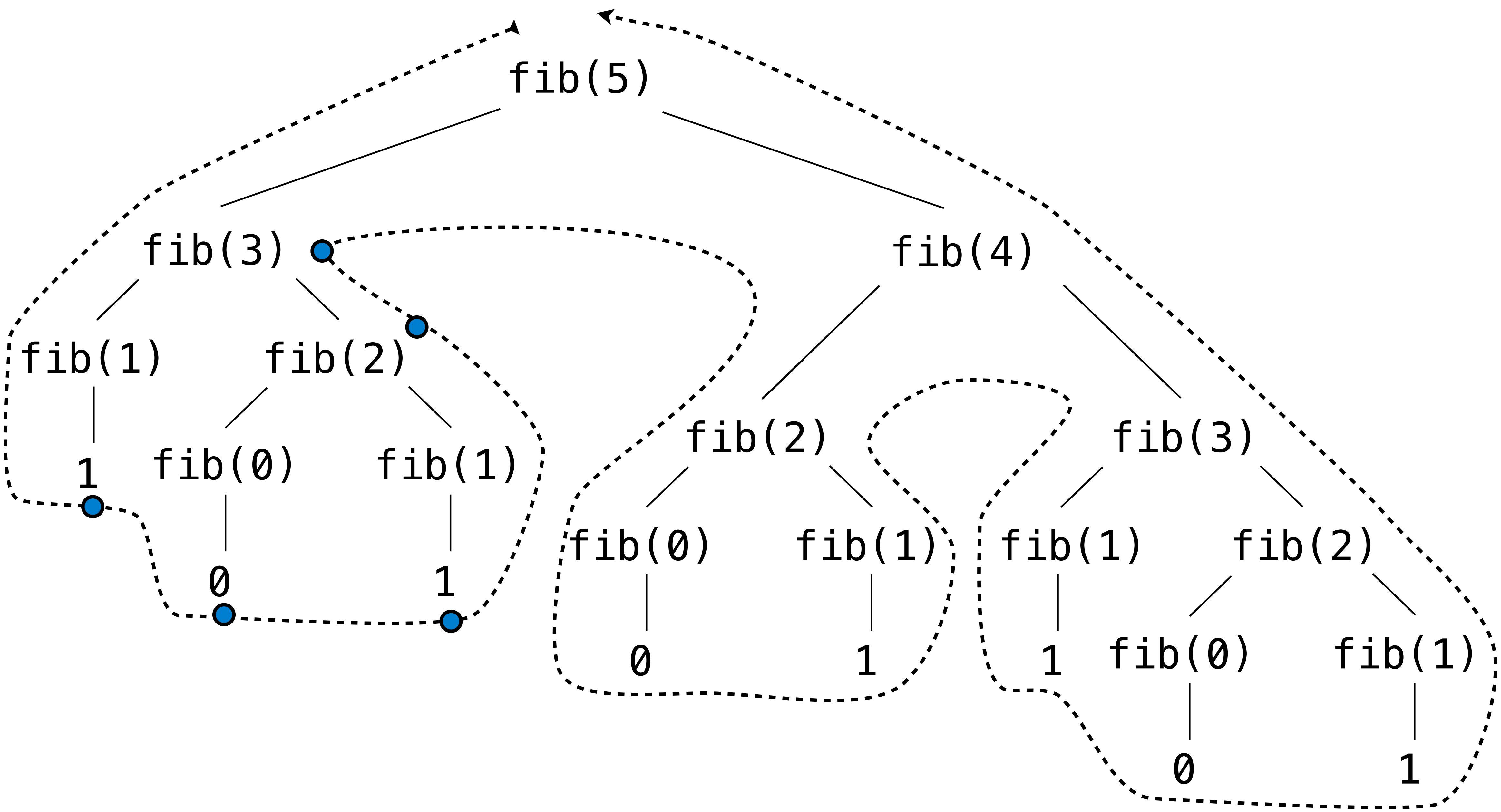

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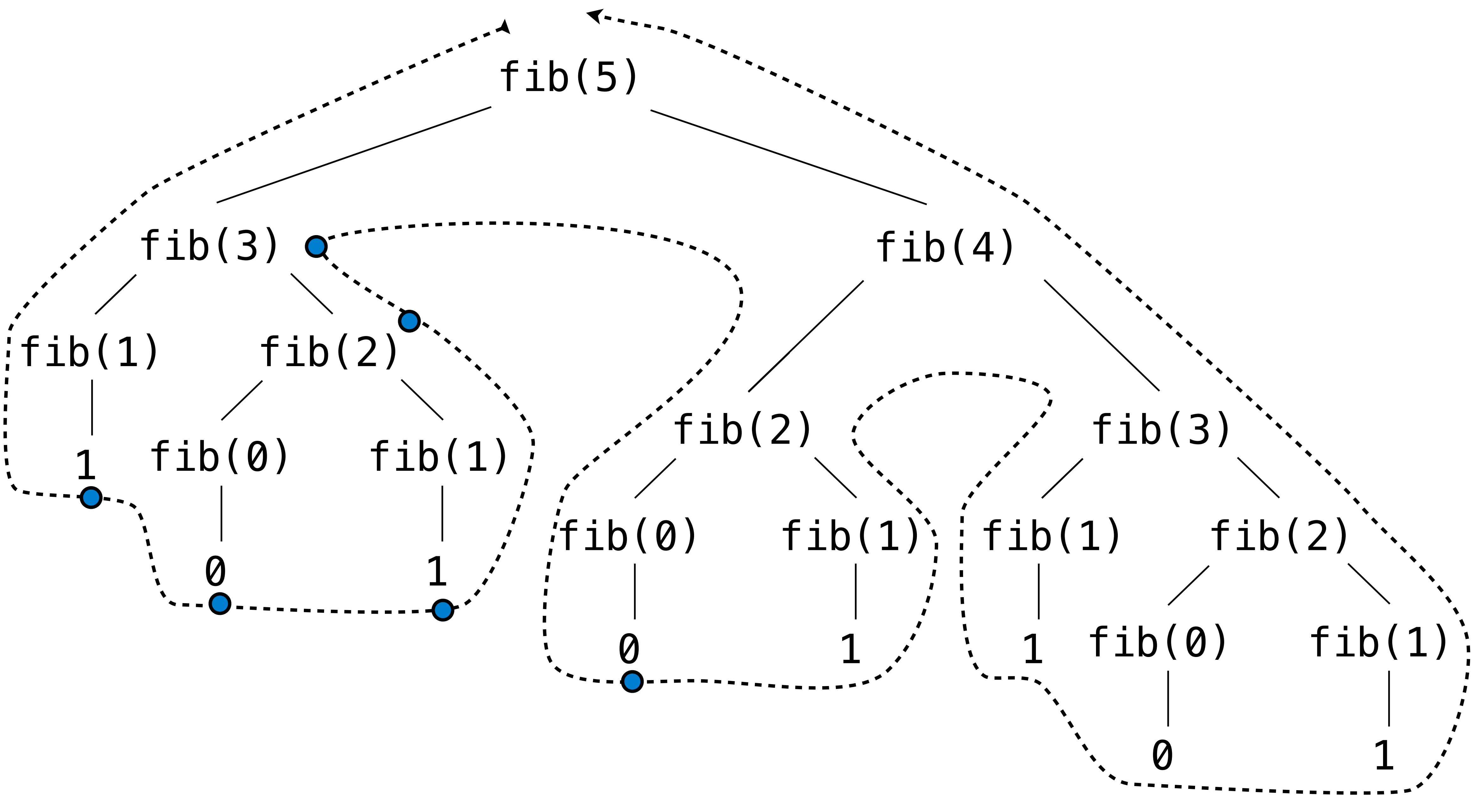

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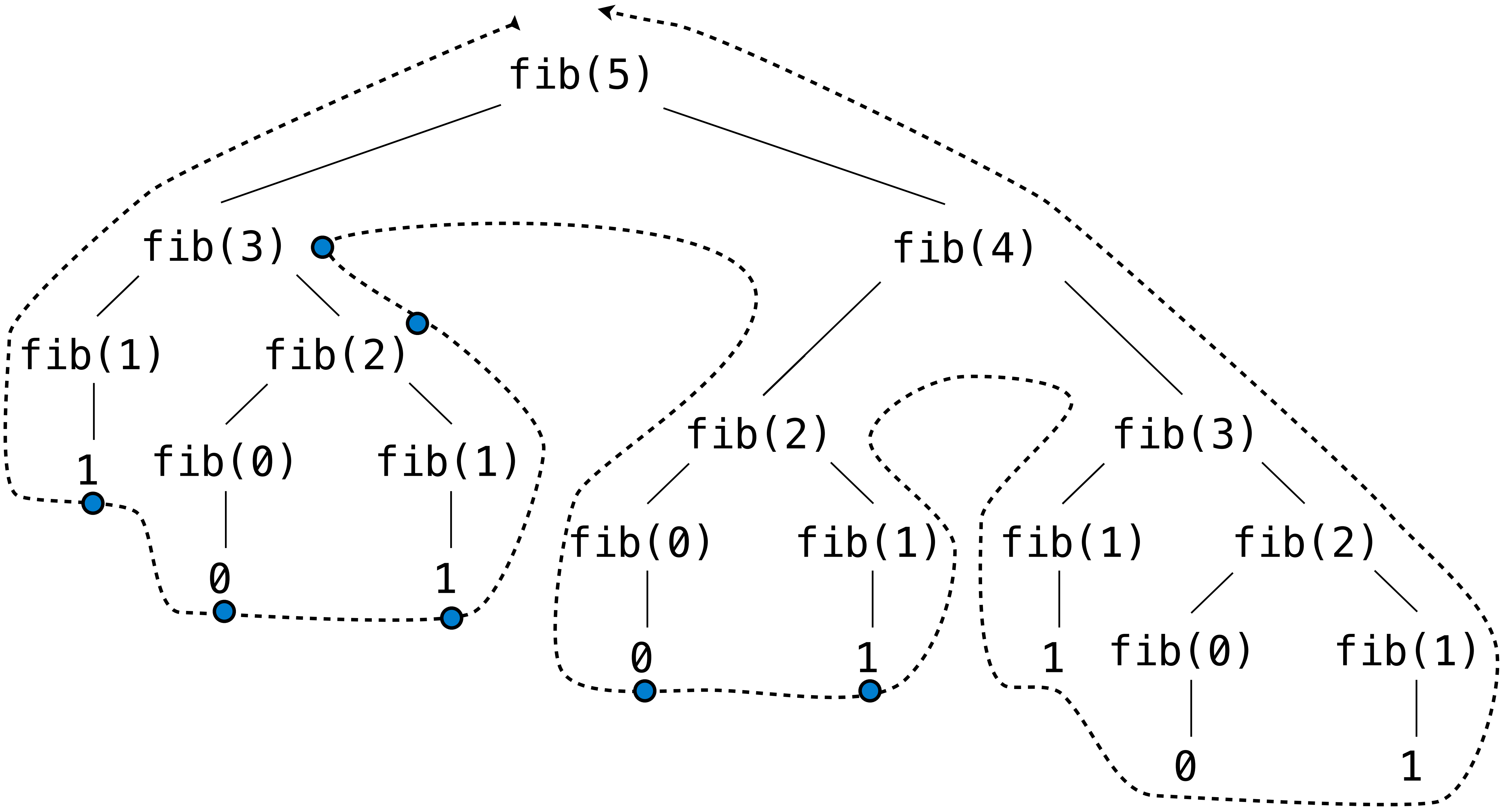

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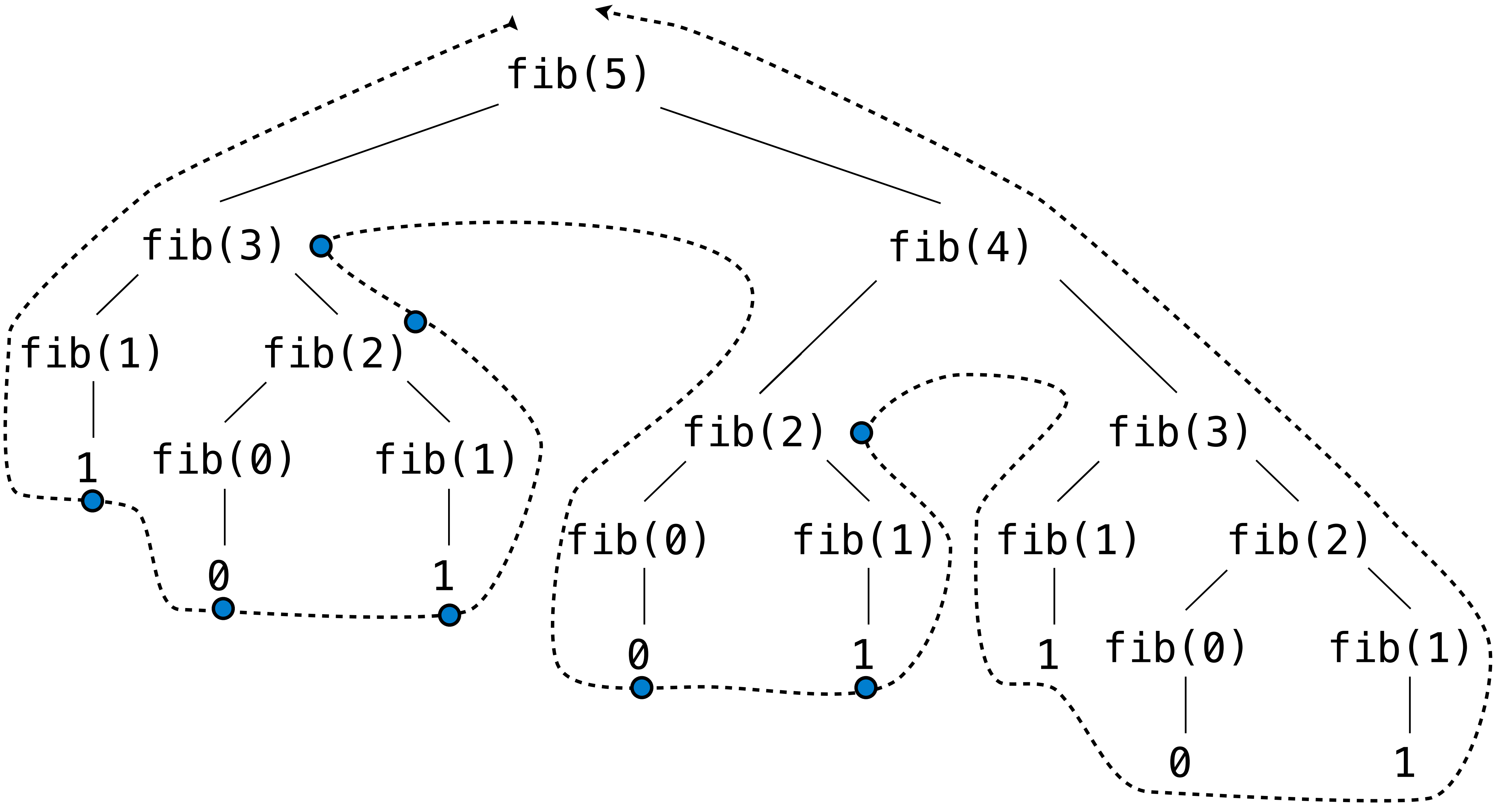

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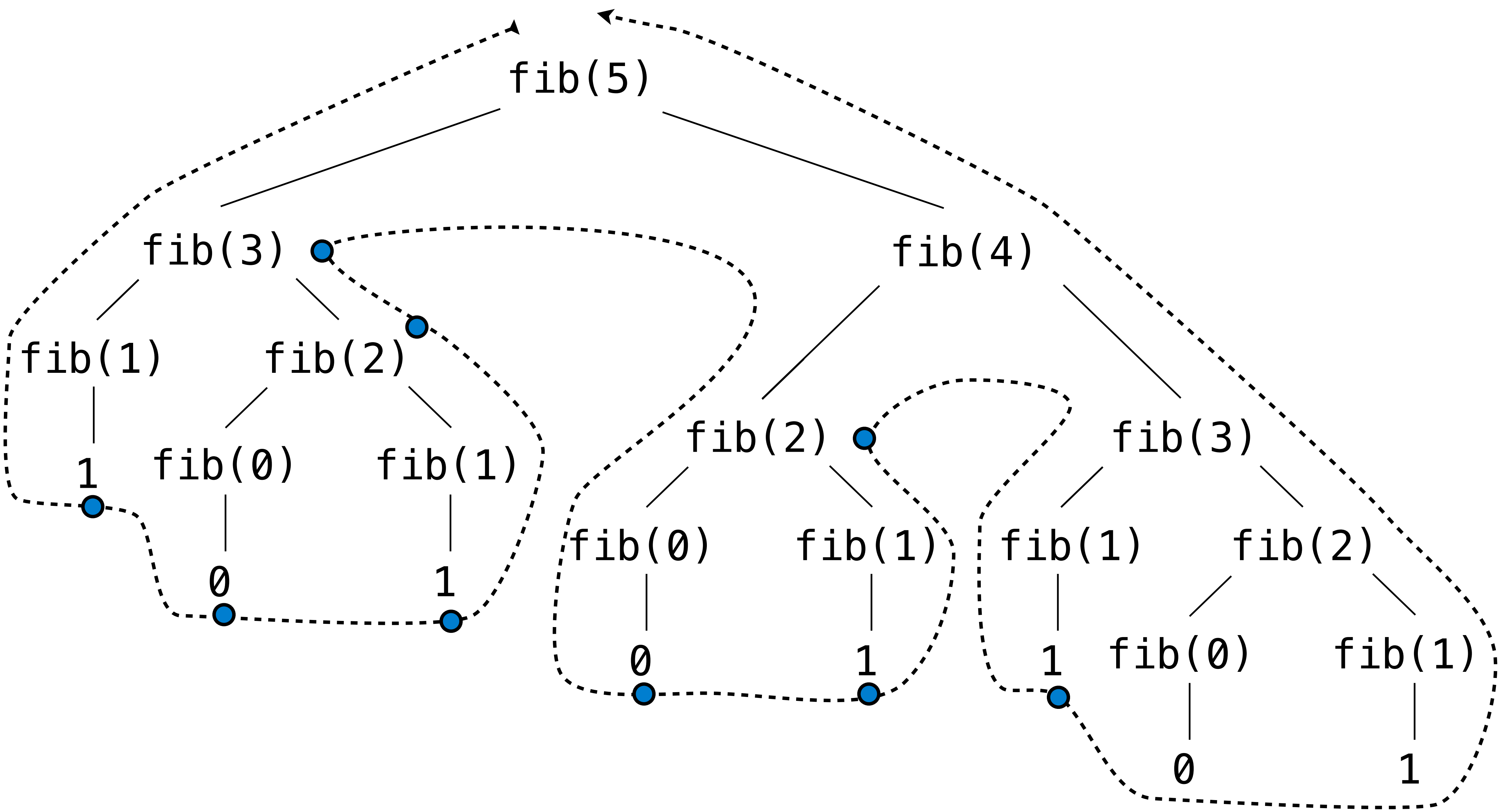

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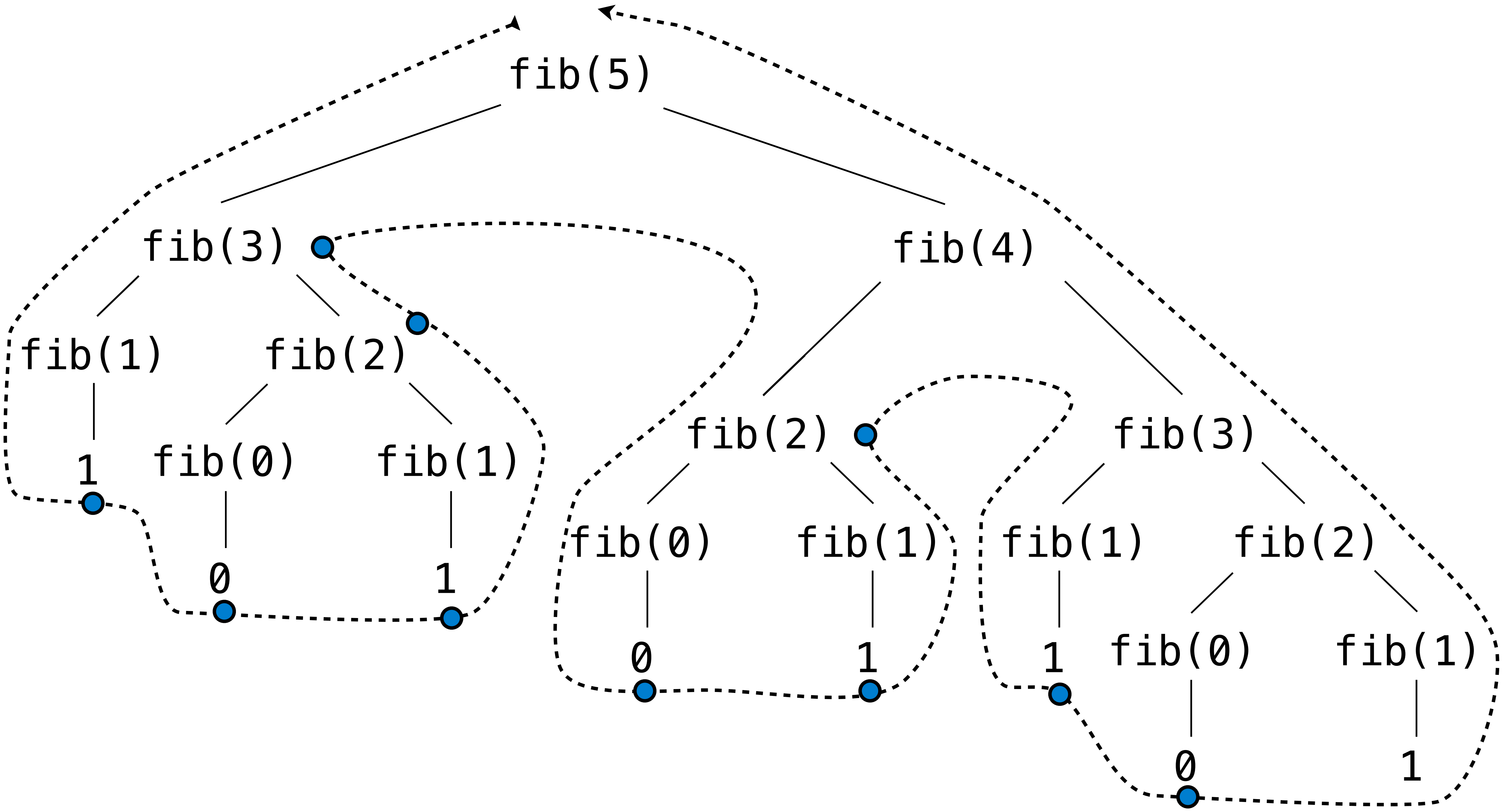

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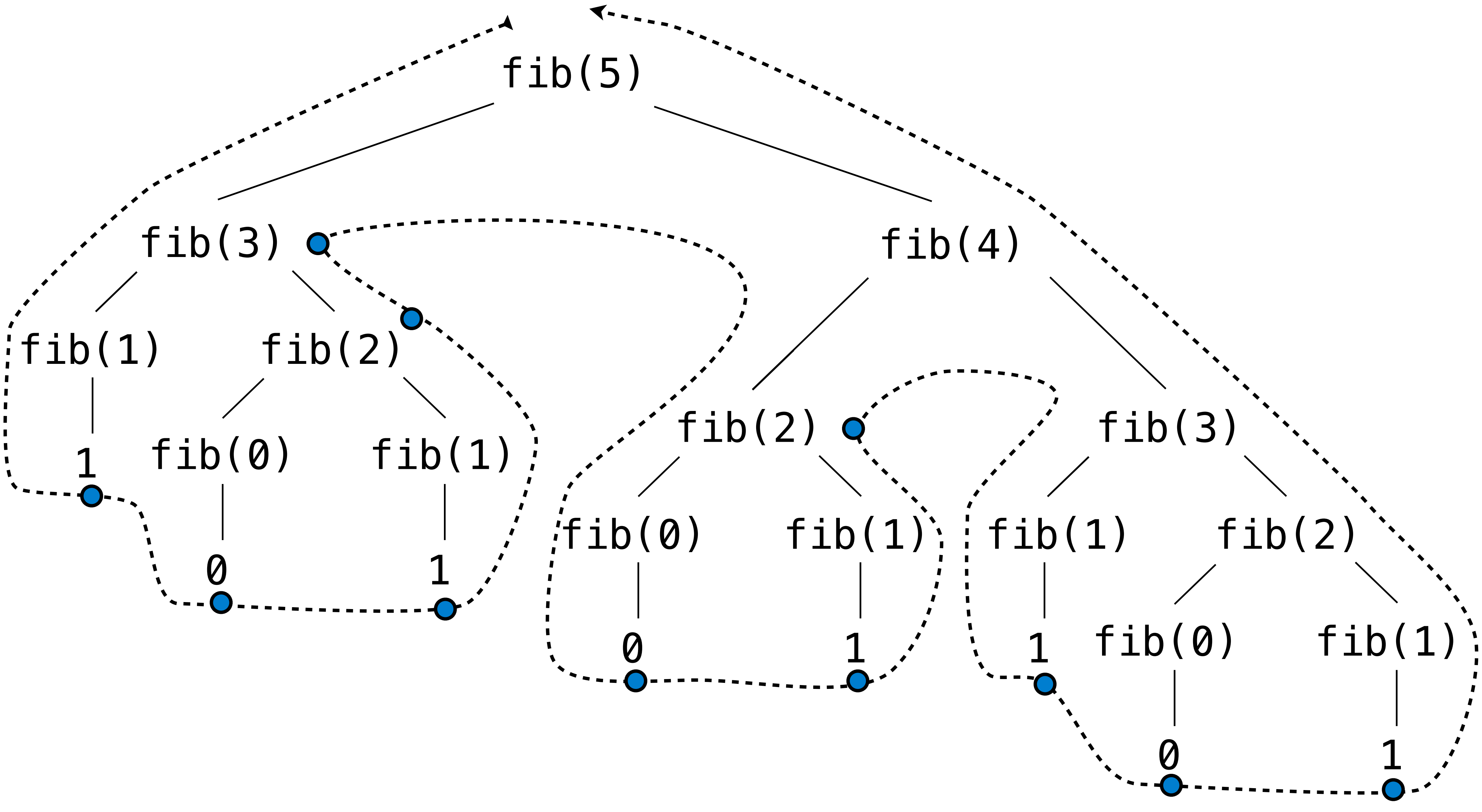

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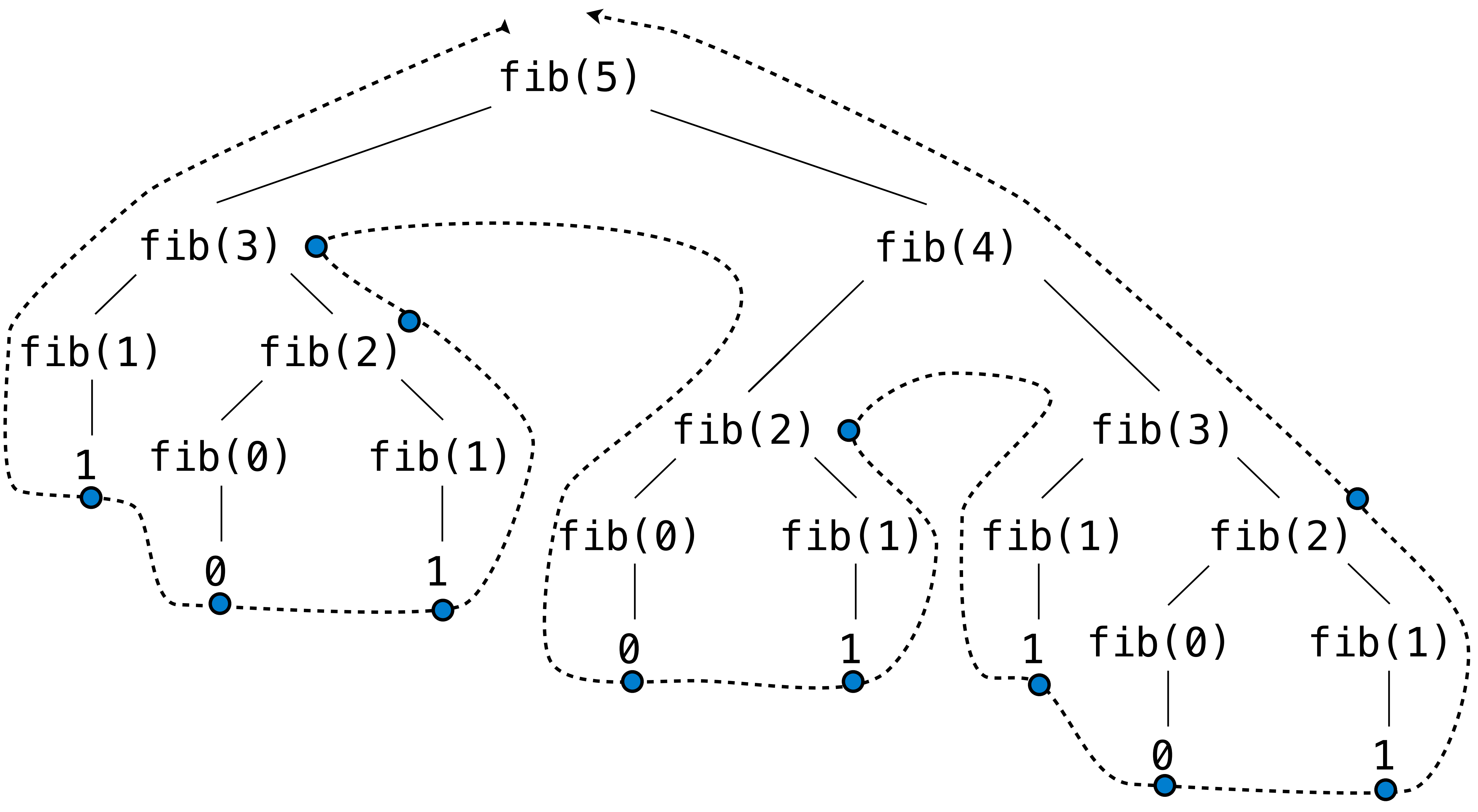

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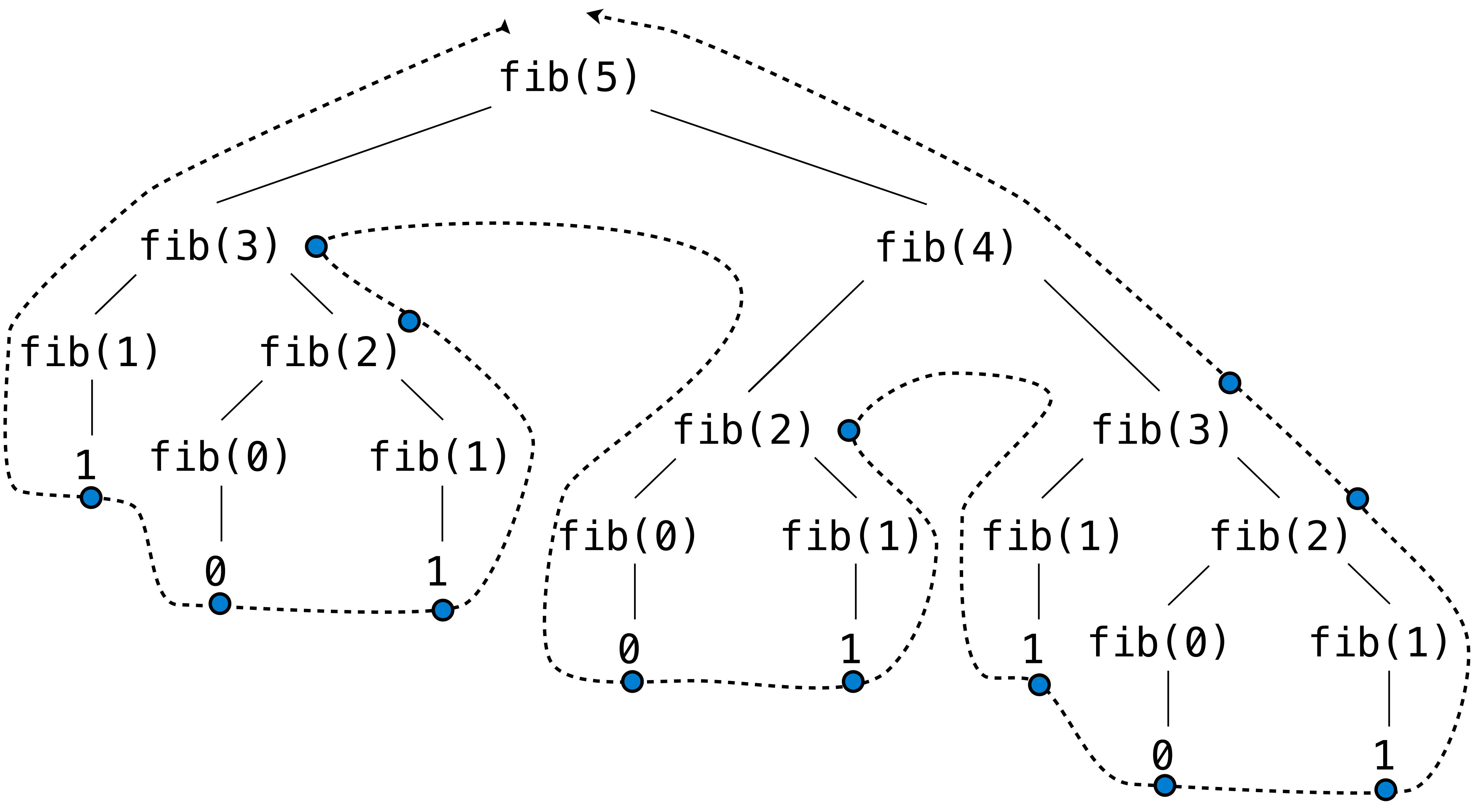

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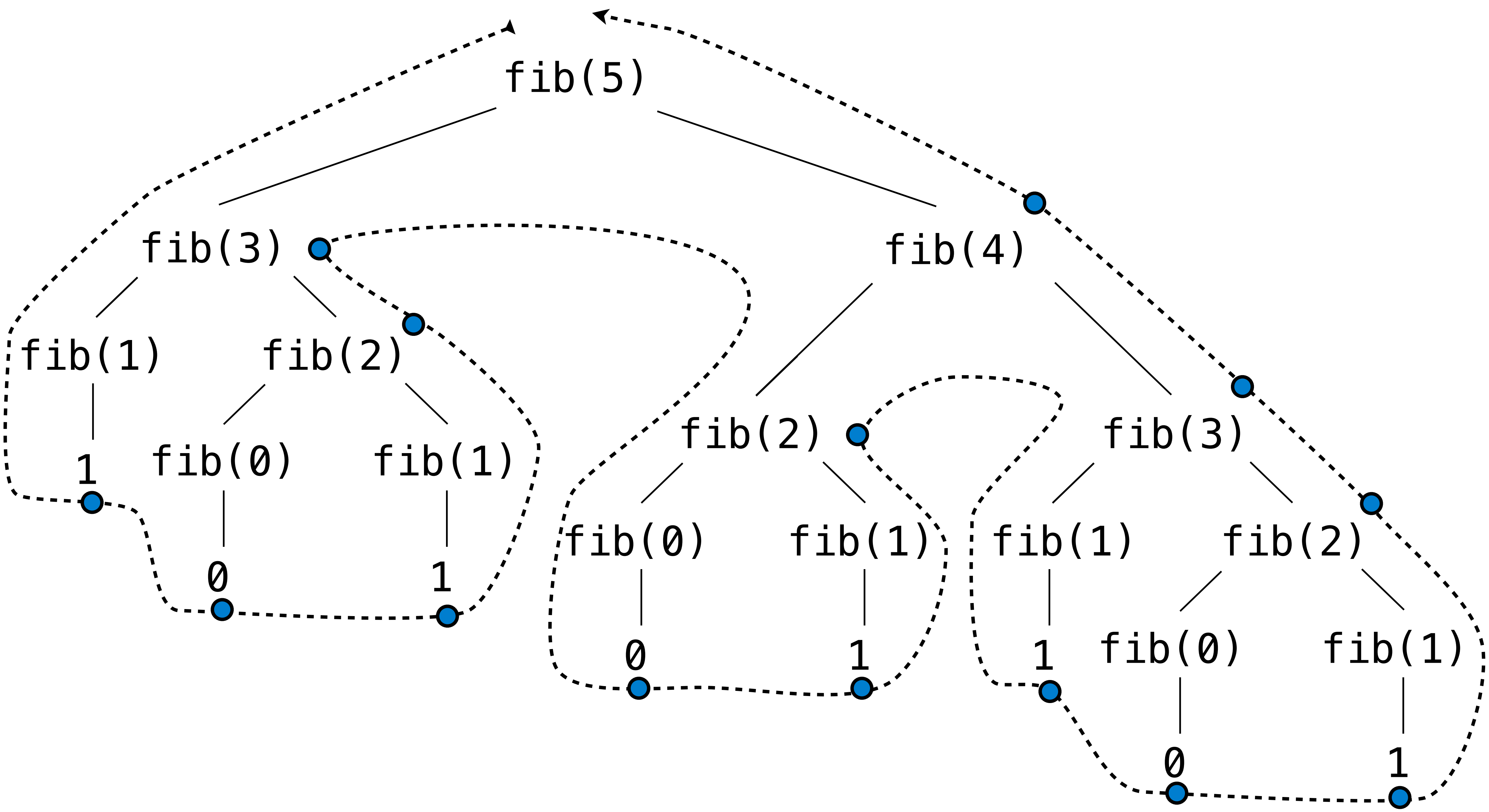

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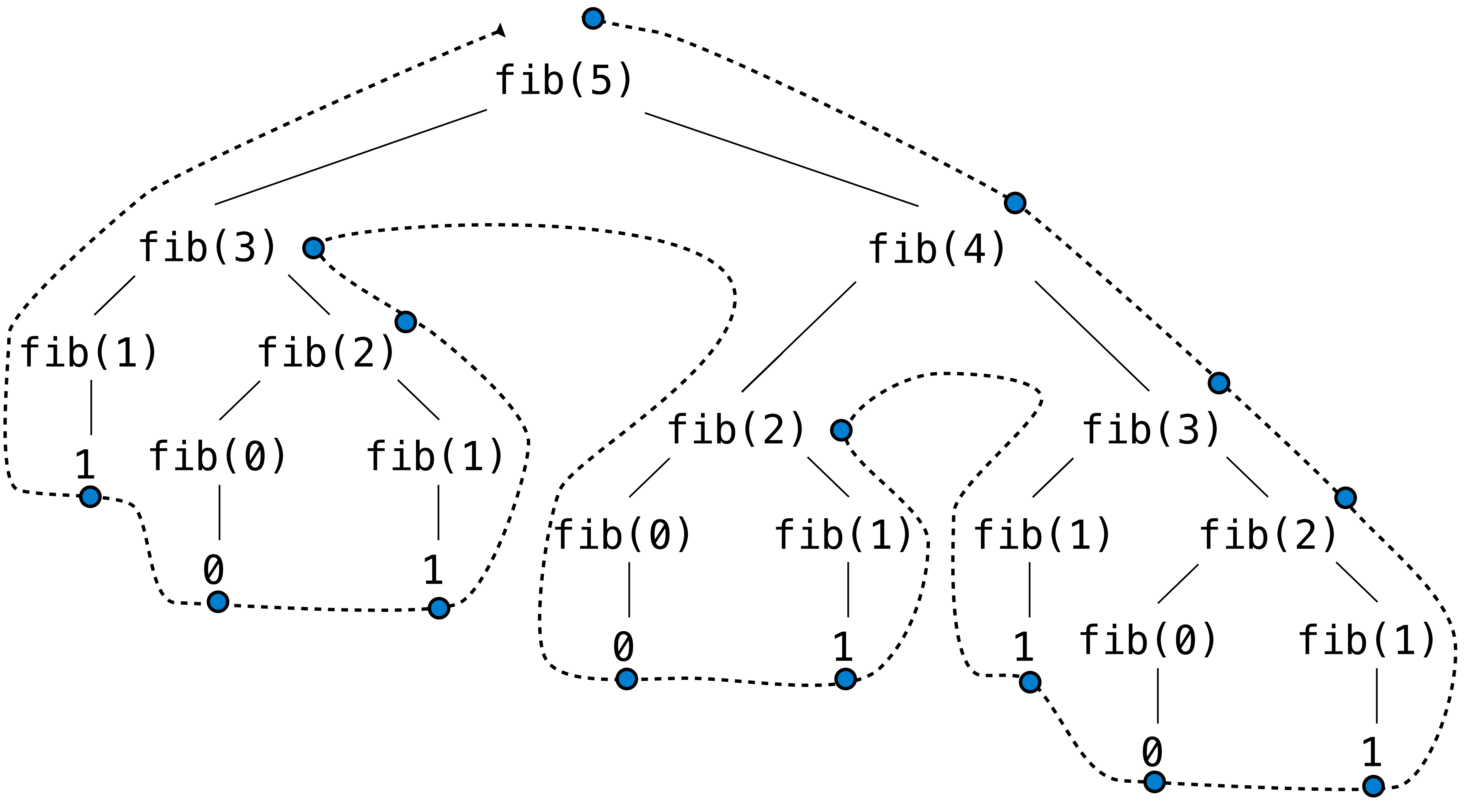
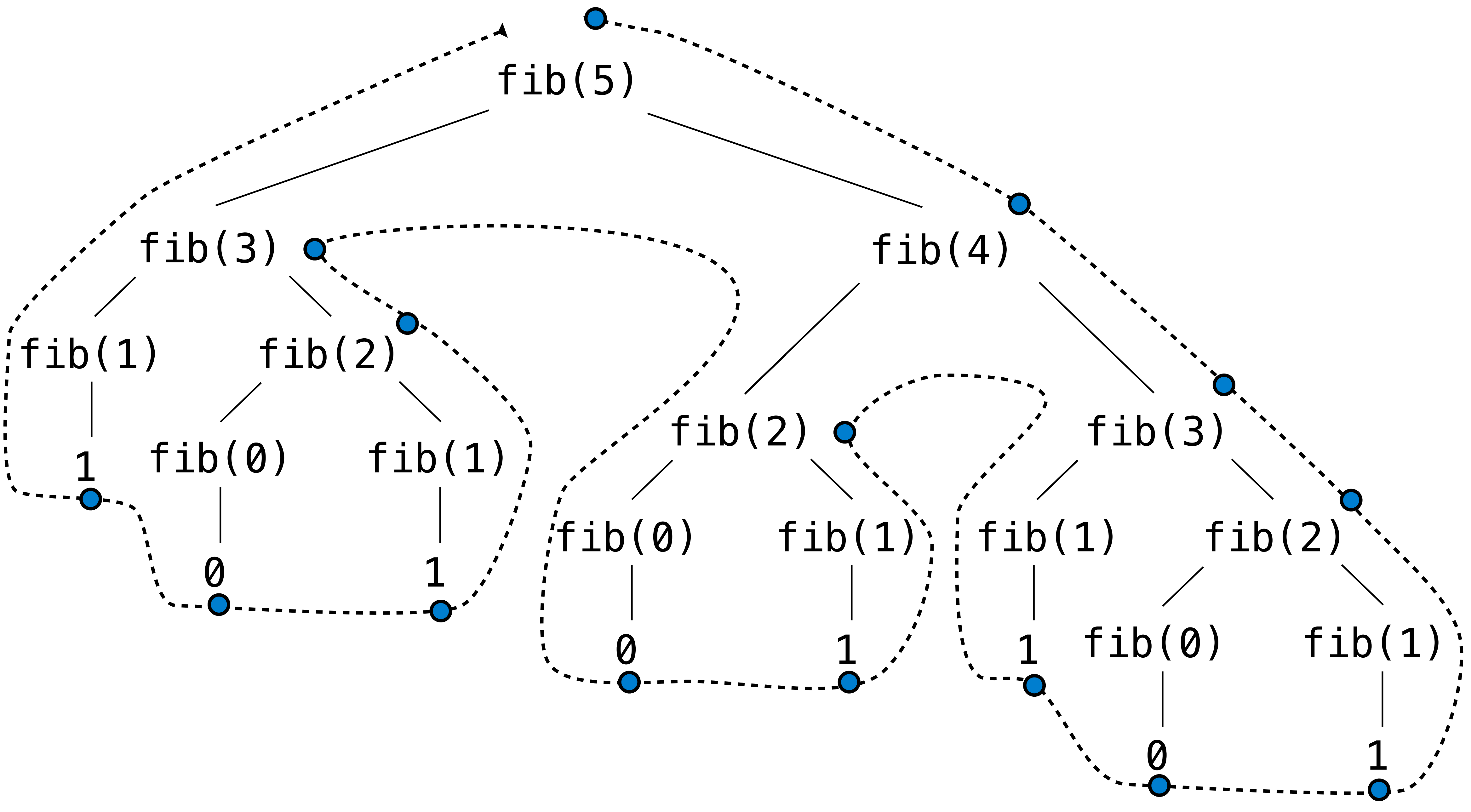
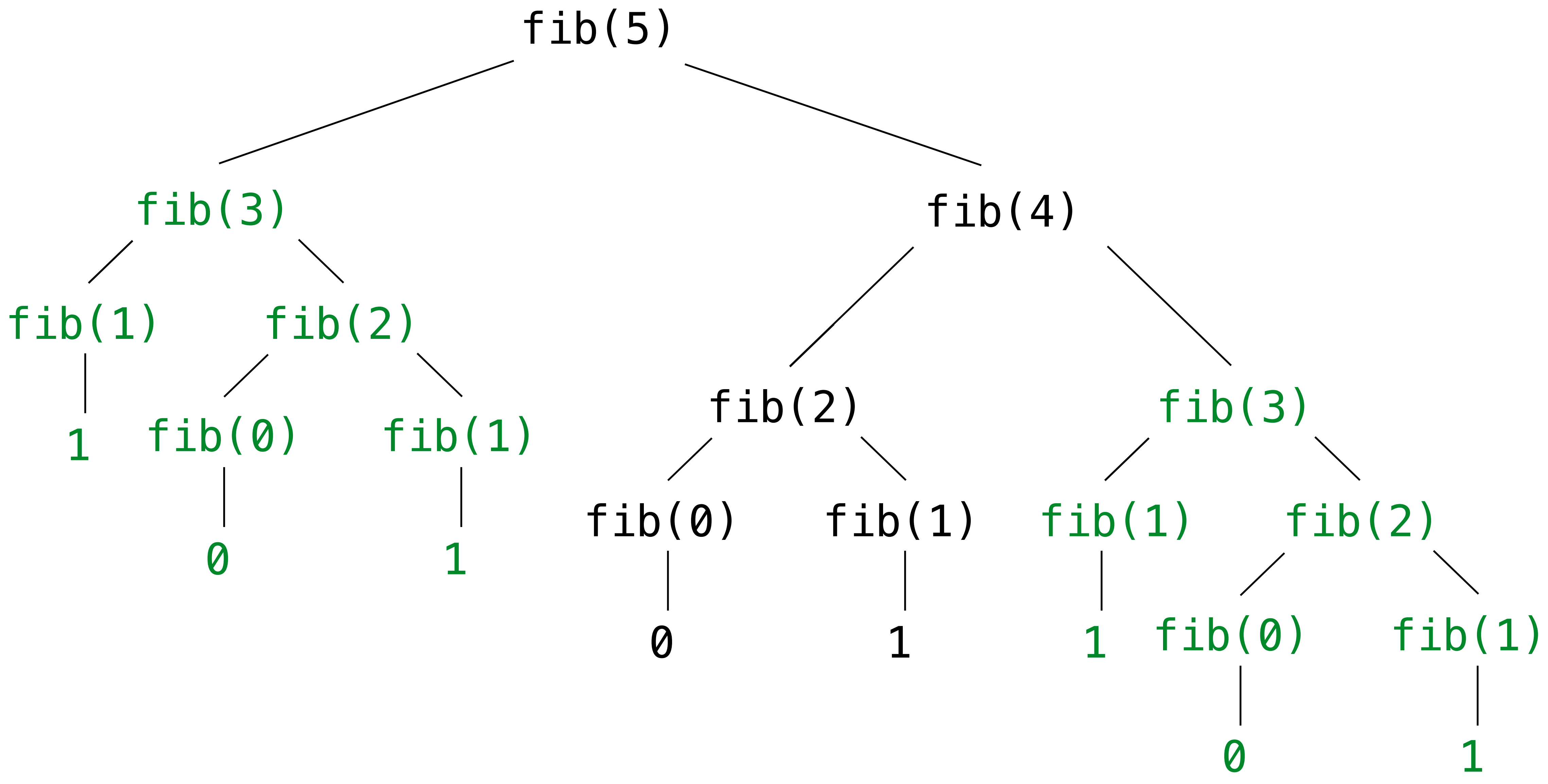


# A Tree-Recursive Process (demo)



# A Tree-Recursive Process



# Break!

# Counting Partitions

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The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

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**count_partitions(6, 4)**


How many different ways can I give out 6 pieces of chocolate if nobody can have more than 4 pieces?

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**count_partitions(6, 4)**


How many different ways can I give out 6 pieces of chocolate if nobody can have more than 4 pieces?

2 + 4 = 6
1 + 1 + 4 = 6

3 + 3 = 6
1 + 2 + 3 = 6
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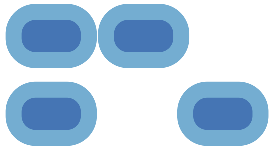

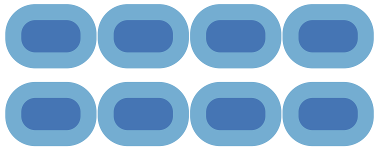

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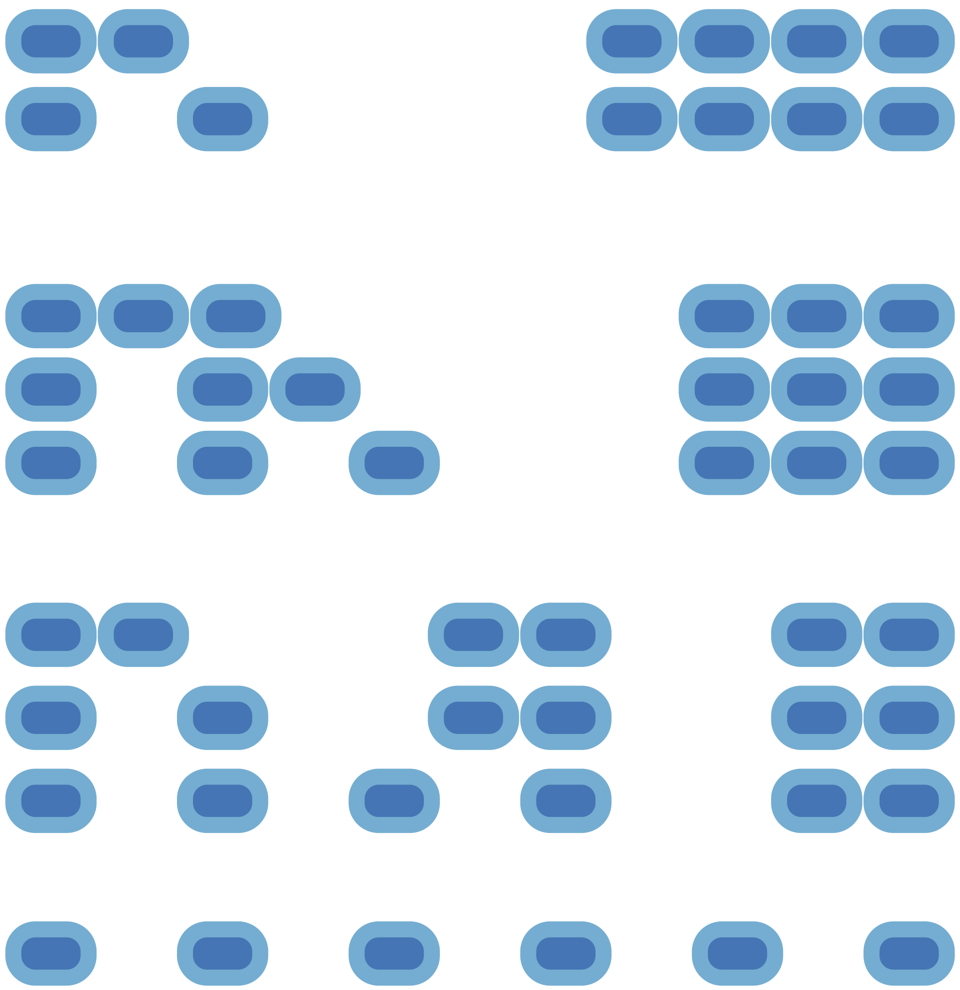

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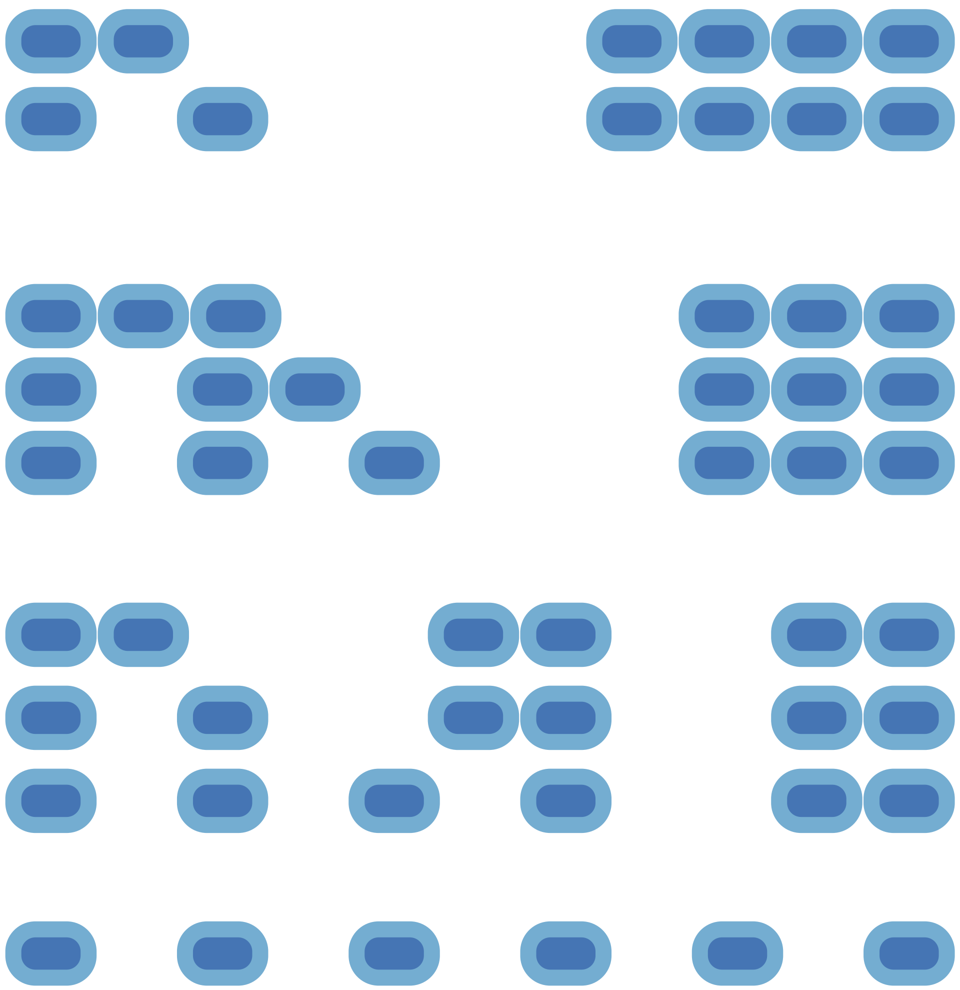

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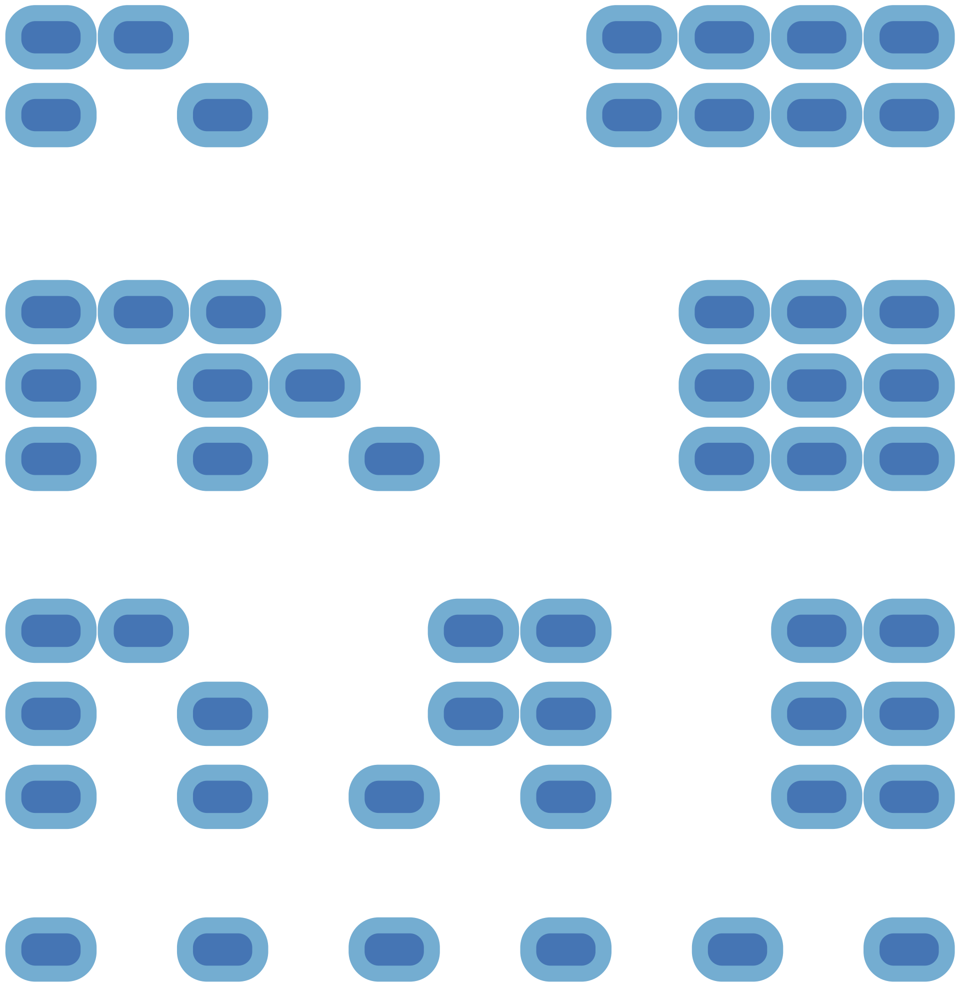

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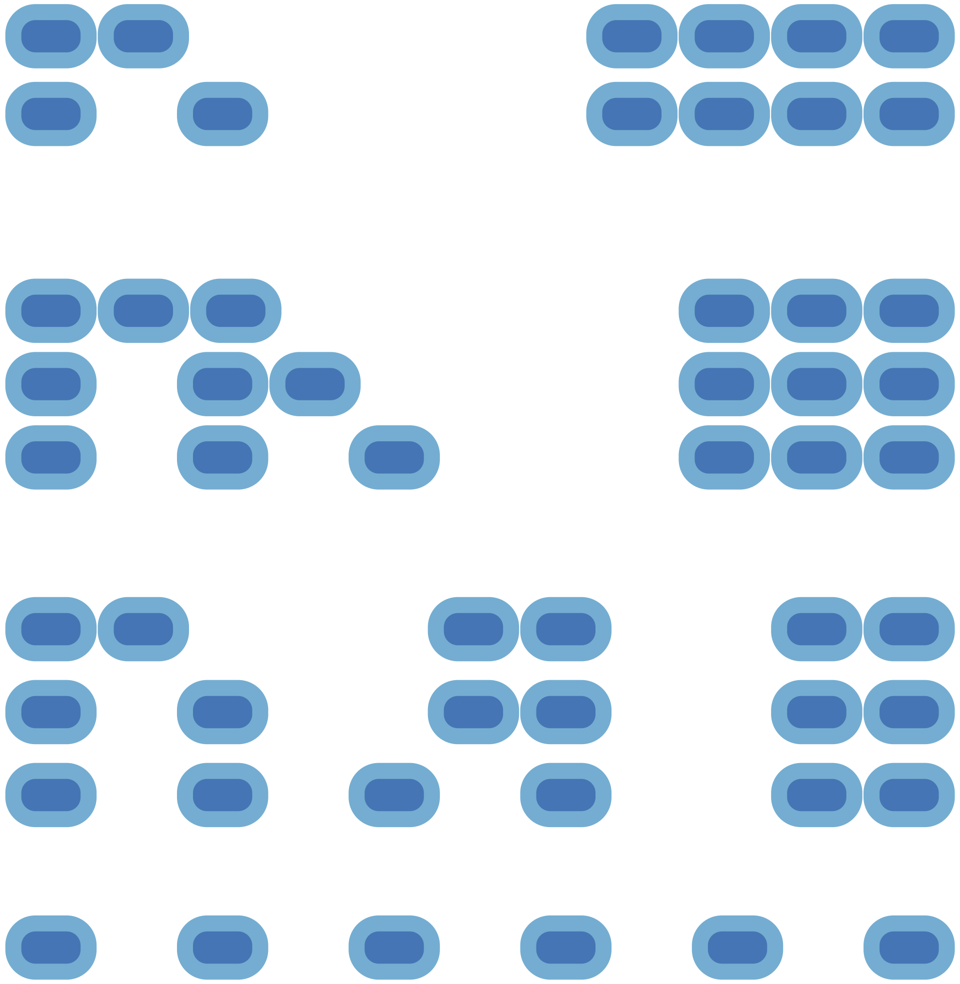

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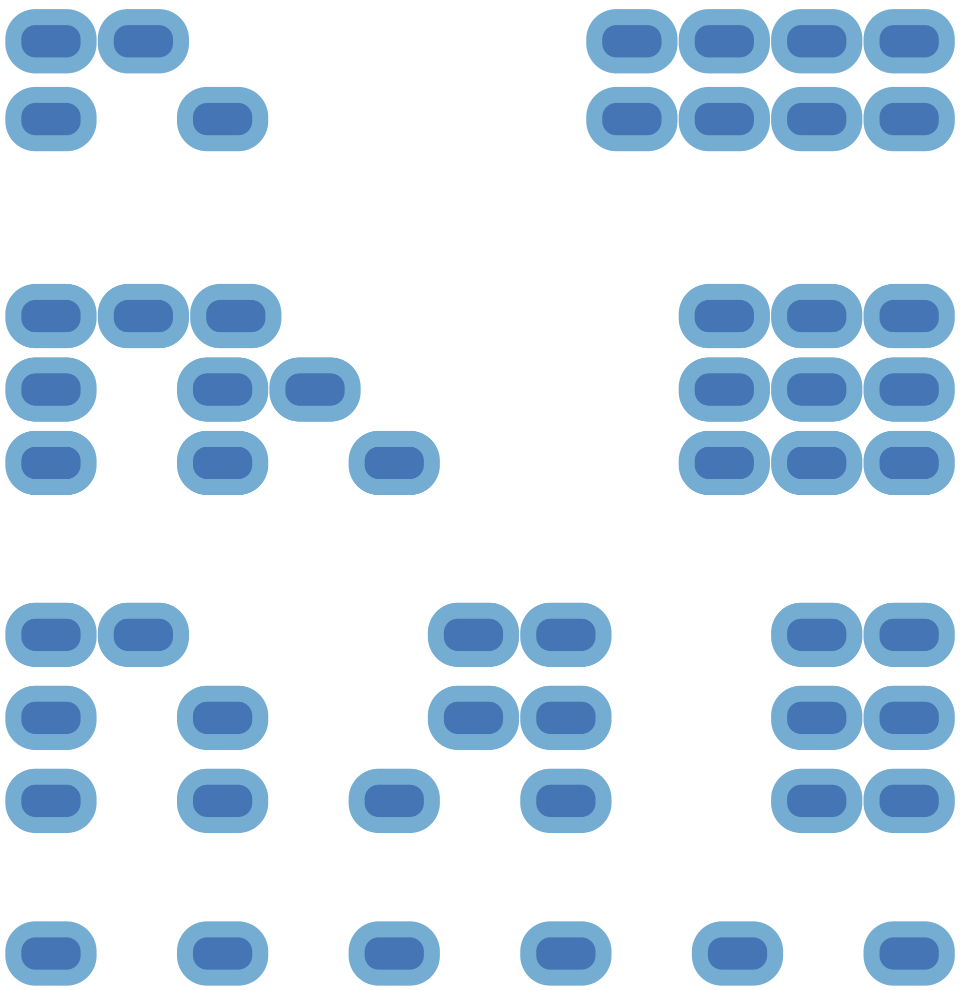

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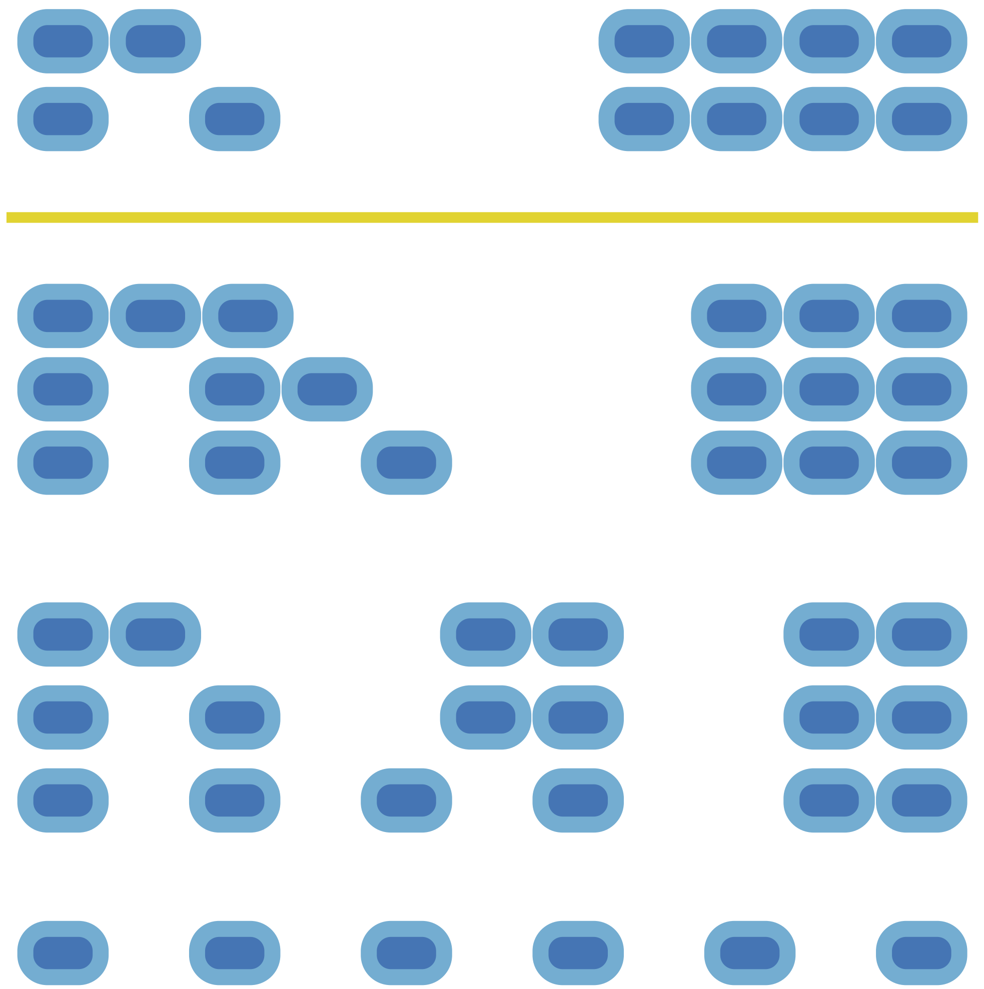

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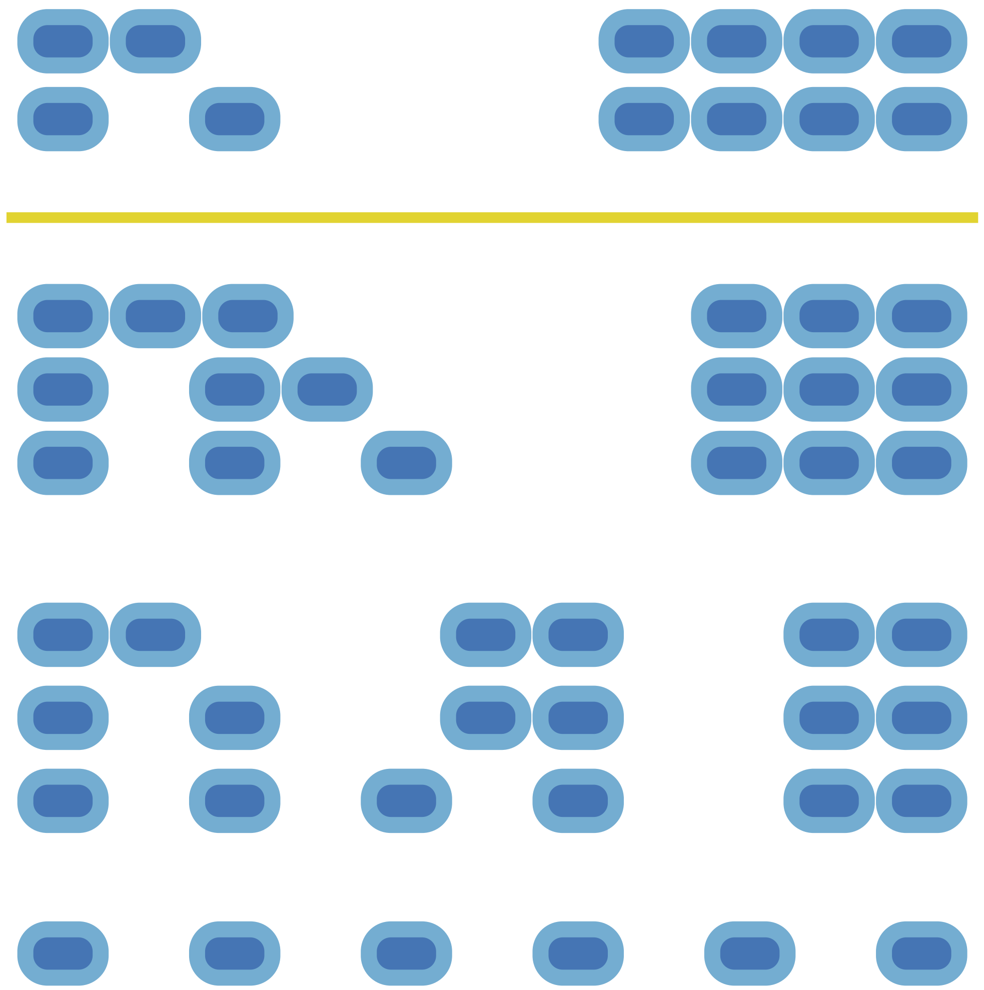
- Explore two possibilities:
  - Use at least one 4



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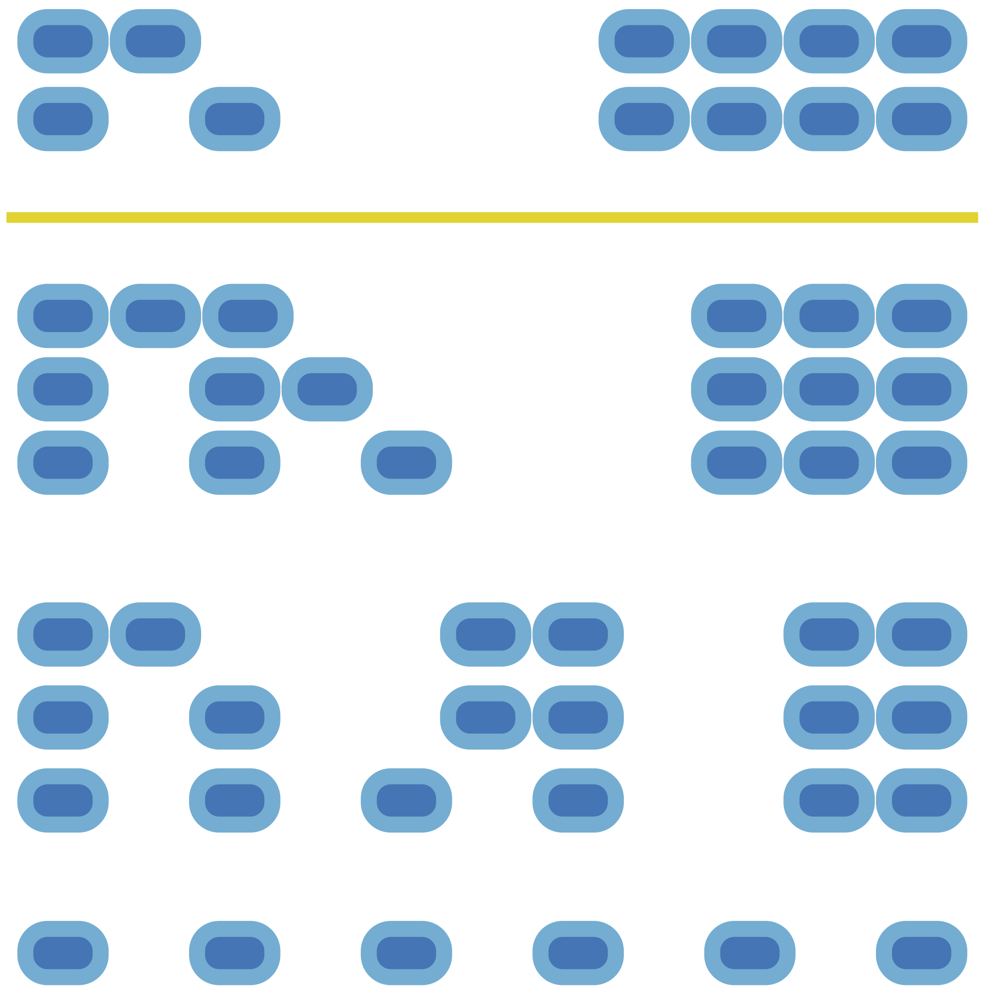

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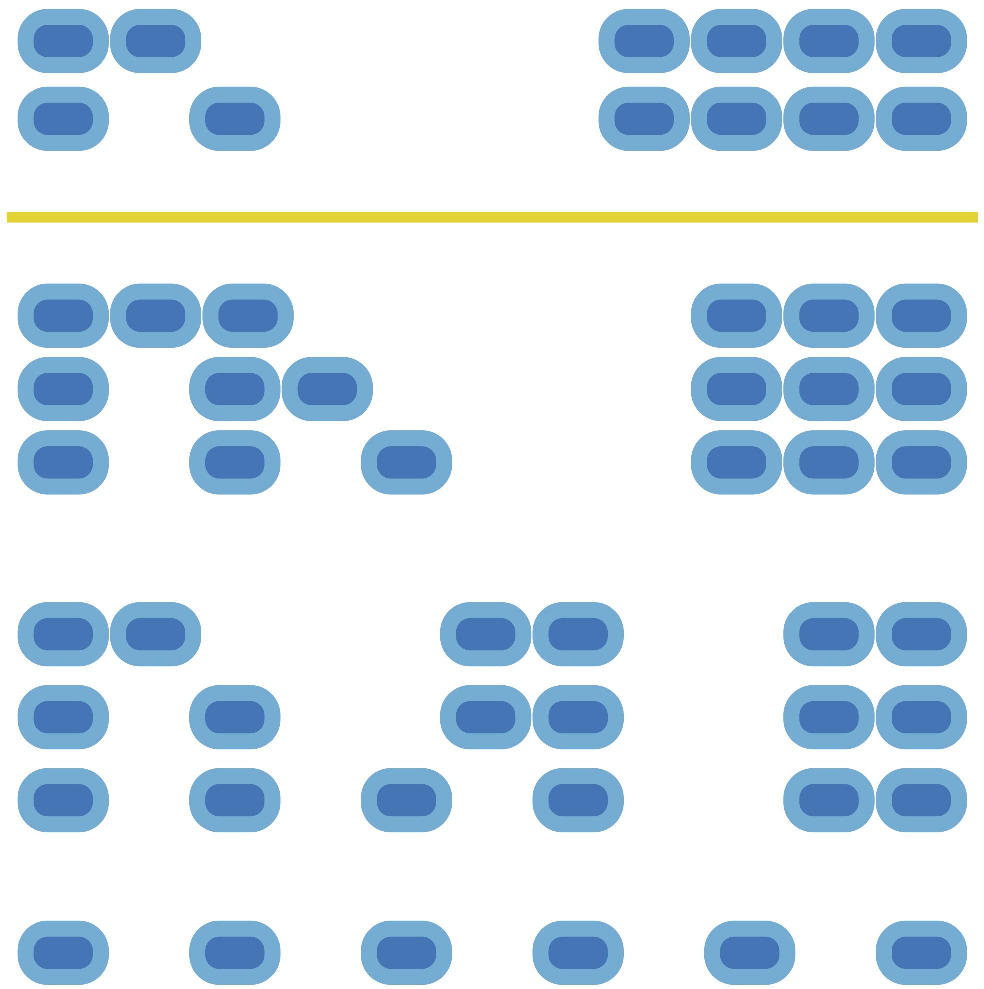

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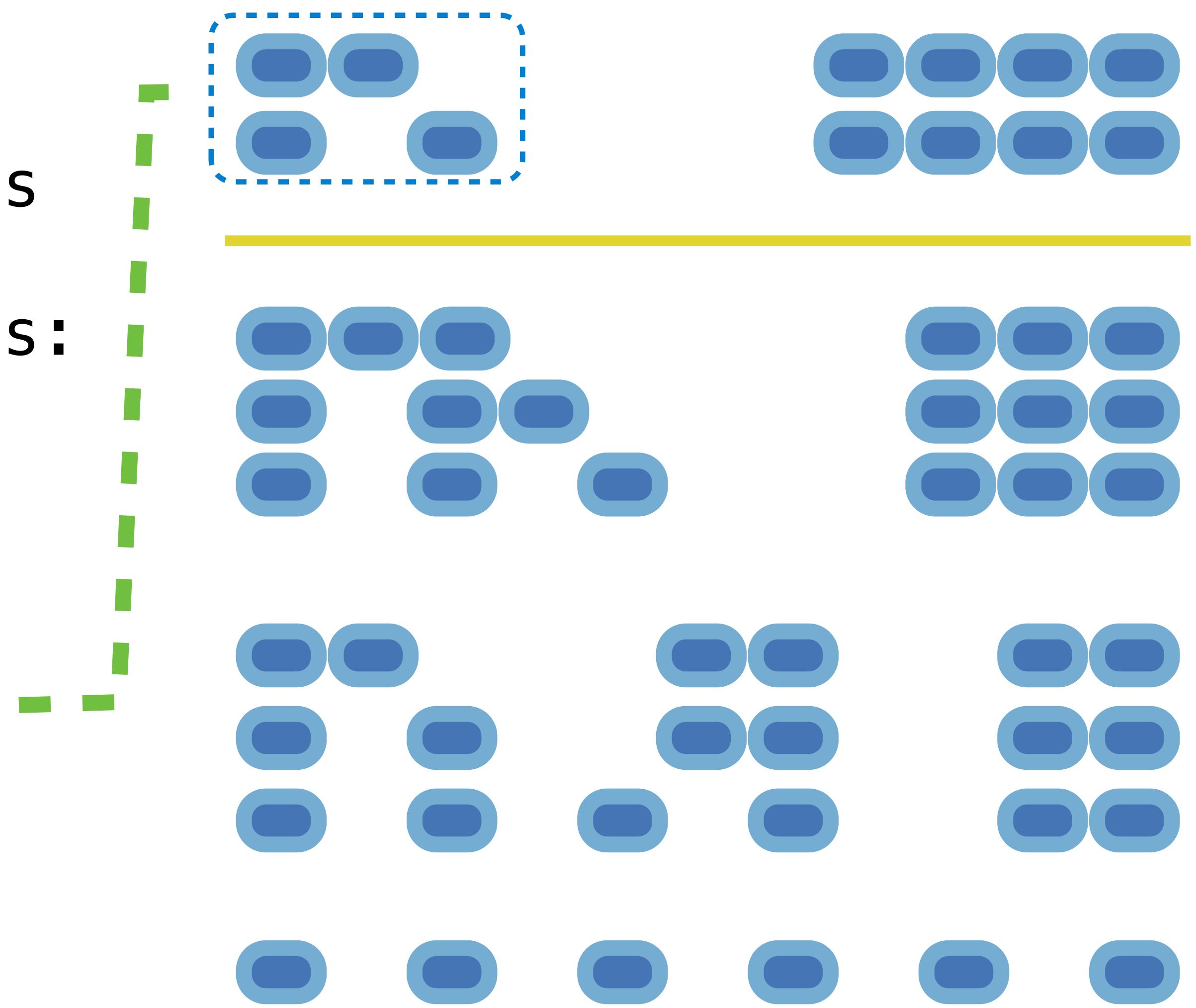
  - count_partitions(2, 4)



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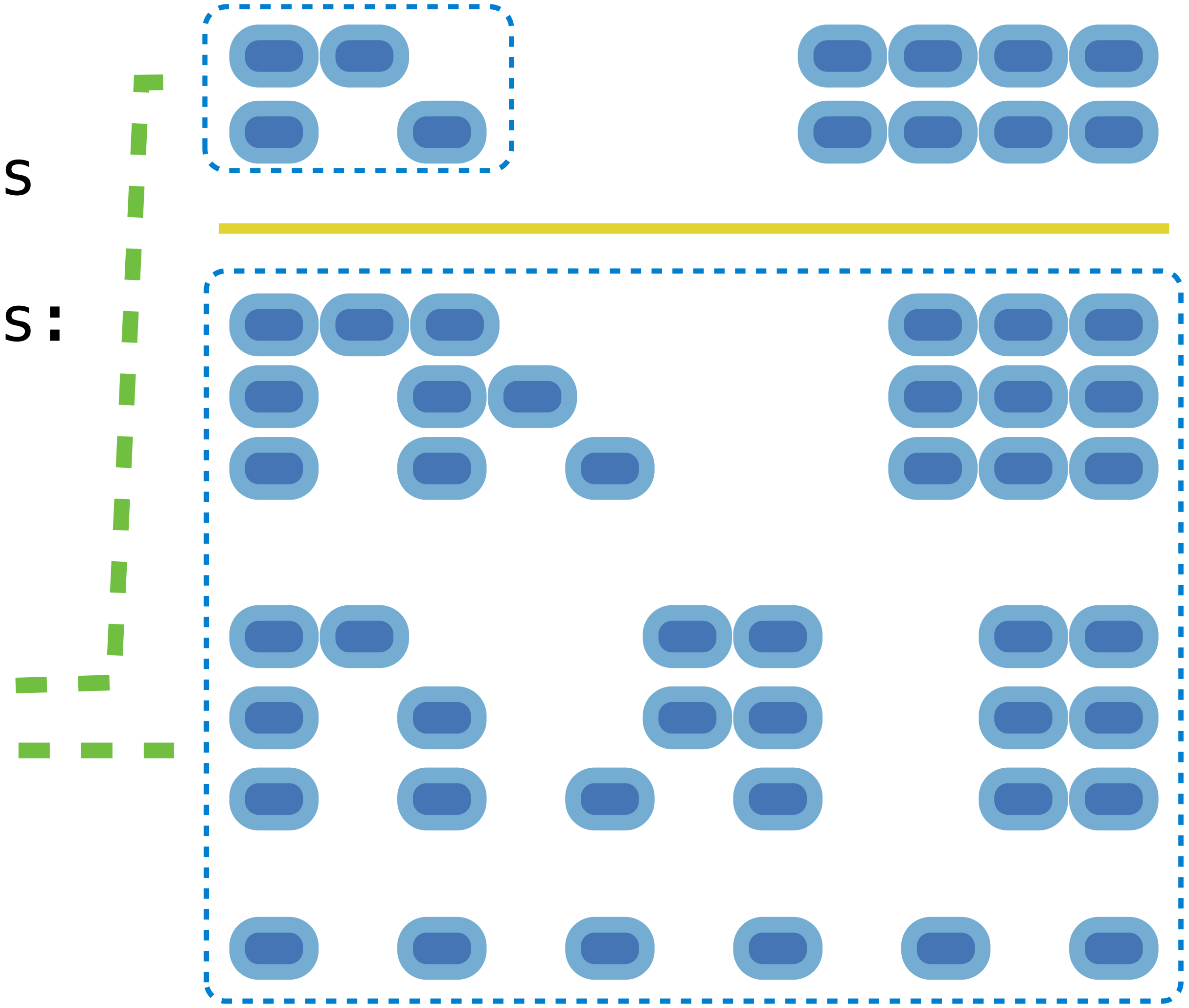
  - count_partitions(2, 4)
  - count_partitions(6, 3)



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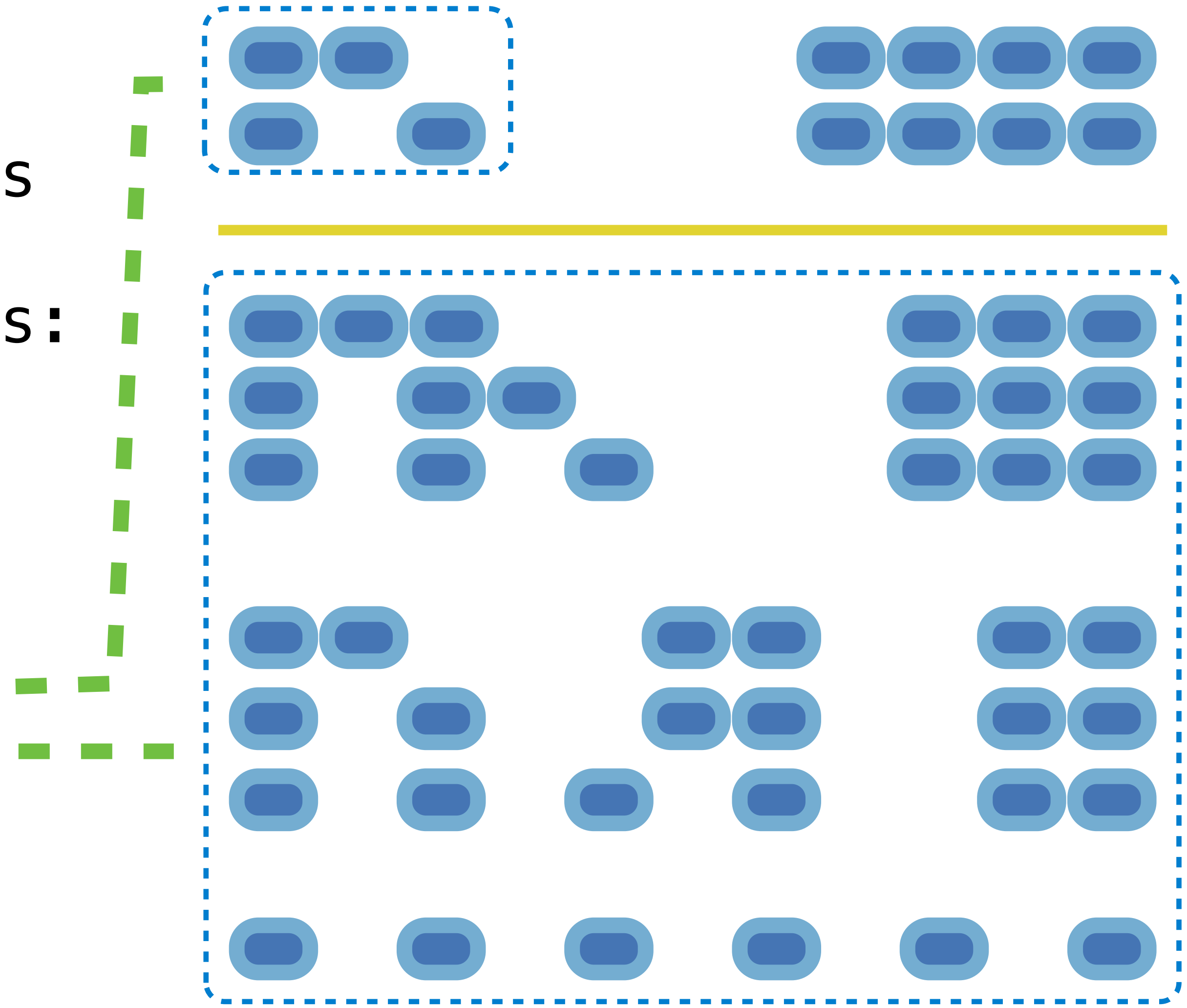
- Tree recursion often involves exploring different choices.



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with_m = count_partitions(n-m, m)
```

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```
with_m = count_partitions(n-m, m)
without_m = count_partitions(n, m-1)
```

# Counting Partitions

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def count_partitions(n, m):
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```
    with_m = count_partitions(n-m, m)
    without_m = count_partitions(n, m-1)
    return with_m + without_m
```

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```
def count_partitions(n, m):
    if n == 0:



        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
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- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1



        with_m = count_partitions(n-m, m)
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- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:



        with_m = count_partitions(n-m, m)
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```

# Counting Partitions

The number of partitions of a positive integer n, using parts up to size m, is the number of ways in which n can be expressed as the sum of positive integer parts up to m in increasing order.

- Recursive decomposition: finding simpler instances of the problem.
- Explore two possibilities:
  - Use at least one 4
  - Don't use any 4
- Solve two simpler problems:
  - count_partitions(2, 4)
  - count_partitions(6, 3)
- Tree recursion often involves exploring different choices.

```
def count_partitions(n, m):
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:


        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m
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