Lecture 6: Recursion

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Announcements

- Hog is due Thursday! Submit Wednesday for 1 EC point
  - Be sure to run `--submit` to check against hidden tests
- HW2 is due Wednesday! Submit Wednesday for credit
- Tutors have begun small tutoring sessions!
  - Check Piazza for details
- Starting this week, lab assistants are running checkoffs in lab sections!
  - Talk to a lab assistant for a few minutes about your lab or homework assignment
  - http://cs61a.org/articles/about.html#checkoffs
- Quiz 2 is *this Thursday*
- Alternate Exam Request: goo.gl/forms/FDQix4I5dNXPQDgw2

Roadmap

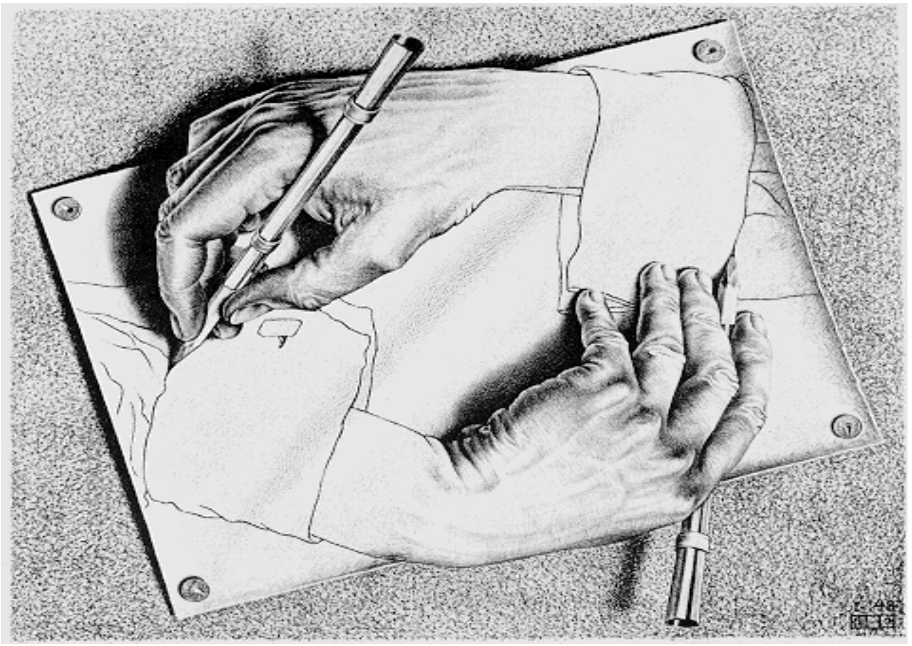
- Introduction
- Functions
- Data
- Mutability
- Objects
- Interpretation
- Paradigms
- Applications

- This week (Functions), the goals are:
  - To understand the idea of *functional abstraction*
  - To study this idea through:
    - higher-order functions
    - recursion (today and tomorrow!)
    - orders of growth

Recursion

- A function is *recursive* if the body of that function contains a call to itself
  - This implies that executing the body of a recursive function may require applying that function

- How is this possible? We'll see some examples next.



Recursion (demo)

- Why would we want to do this?
  - A common problem solving technique is to *break down the problem* into smaller problems that are easier to solve
  - This is exactly what recursion does!
  - For example, how would you write a function that, given a string, returns the reversed version of the string?

Anatomy of a Recursive Function

- The def statement header is similar to other functions
- Conditional statements check for base cases
- Base cases are evaluated without recursive calls
- Recursive cases are evaluated with recursive calls

```
def factorial(n):
    """Return the factorial of n."""
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)
```

Verifying Correctness

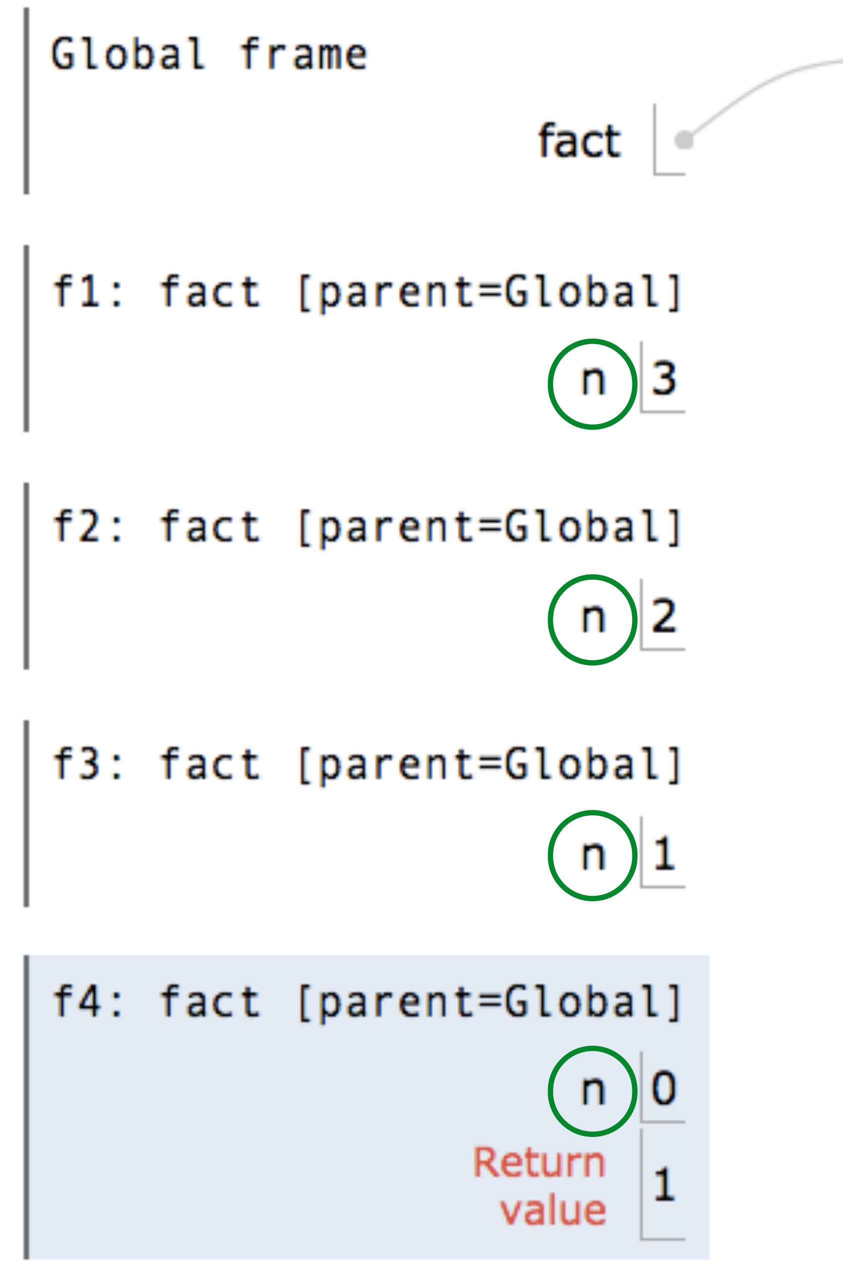
The easy way, and the right way

Recursion in Environment Diagrams (demo)

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)

fact(3)
```

- The same function `fact` is called multiple times
- Different frames keep track of the different arguments in each call
- What `n` evaluates to depends upon the current environment
- Each call to `fact` solves a simpler problem than the last: smaller `n`



Better: the Recursive Leap of Faith

```
def factorial(n):
    """Return the factorial of n."""
    if n == 0:
        return 1
    else:
        return n * factorial(n-1)
```

Is `factorial` **implemented correctly?**

1. Verify the base case(s).
    1. Are they *correct*?
    2. Are they *exhaustive*?

Now, harness the power of *functional abstraction*!

2. Assume that `factorial(n-1)` is correct.
3. Verify that `factorial(n)` is correct.

Writing Recursion (demo)

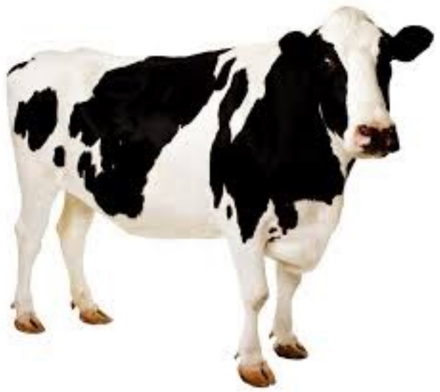
```
def sum_digits(n):
    """Return the sum of the digits of n.

    >>> sum_digits(2016)
    9
    """
```

```
if n < 0:
    return 0
```

```
if n == 1:
    return 1
```

```
if n < 10:
    return n
```



```
if n < 100:
    return n
```

Writing Recursion (demo)

```
def sum_digits(n):
    """Return the sum of the digits of n.

    >>> sum_digits(2016)
    9
    """
    if n < 10:
        return n
    else:
        return sum_digits(n//10) + n%10
```

Iteration vs Recursion (demo)

- Iteration is a special case of recursion
- Converting iteration to recursion is formulaic, but converting recursion to iteration can be more tricky

Using iteration:

```
def fact_iter(n):
    total, k = 1, 1
    while k <= n:
        total, k = total*k, k+1
    return total
```

Using recursion:

```
def fact(n):
    if n == 0:
        return 1
    else:
        return n * fact(n-1)
```

Math: $$n! = \prod_{k=1}^{n} k$$

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{otherwise} \end{cases}$$

Names: `n, total, k, fact_iter` `n, fact`

Recursion on Sequences (demo)

- We've seen iteration as one way of working with sequences, but iteration is a special case of recursion
- This means that we can also use recursion to solve problems involving sequences!

```
def reverse(word):
    """Return the reverse of the string word."""
    if len(word) < 2:
        return word
    else:
        return reverse(word[1:]) + word[0]
```

Summary

- *Recursive functions* call themselves, either directly or indirectly, in the function body
  - The motivation for this is to *break down* the problem into smaller, easier to solve problems
  - For example, computing the factorial of a smaller number, or the reverse of a shorter string
- Recursive functions have *base cases*, which are not recursive, and *recursive cases*
  - The best way to verify recursive functions is with functional abstraction!
  - Use the *leap of faith*